Emergence of the Infinite:

How the Dynamics of Consciousness Give Birth to the Infinite in the Realm of Sets

PART I : Background

1. The Starting Point for Mathematics: The ZFC Axioms

Every Provable Mathematical Statement Is Derivable from the Set Theory Axioms

- Everything studied in mathematics is a *set*, but what is a set? Attempts to resolve paradoxes led to a set of axioms to answer this question.
- The ZFC axioms tell us basic things that are true about sets. Intuition for the axioms arose both from mathematical practice and from an intuitive model of the universe (the precursor to V)
- Some axioms
  - *Empty Set:* There is a set with no elements
  - *Pairing*: If A and B are sets, {A,B} is also a set
  - *Power Set*: If A is a set, the collection P(A) of all subsets of A is also a set
  - *Infinity*: There is an infinite set

A Complete List

Axioms of ZFC Set Theory

1. (*Empty Set Axiom*) There is a set with no element.

2. (*Axiom of Infinity*) There is an infinite set.

3. (*Axiom of Extensionality*) Two sets are equal if and only if they have the same elements.

4. (*Pairing Axiom*) If X and Y are sets, so is the collection $\{X,Y\}$.

5. (*Union Axiom*) If X is a set whose members are sets, then $\bigcup X$ is also a set.

6. (*Power Set Axiom*) If X is a set, so is $P(X)$, the collection of all subsets of X.

7. (*Axiom of Foundation*) Every nonempty set X has a member y such that no member of y is in X (y is called an $\in$ - *minimal element* of X).

8. (*Axiom of Separation*) For every set X and every property R, the collection of all members of X which satisfy the property R is itself a set.

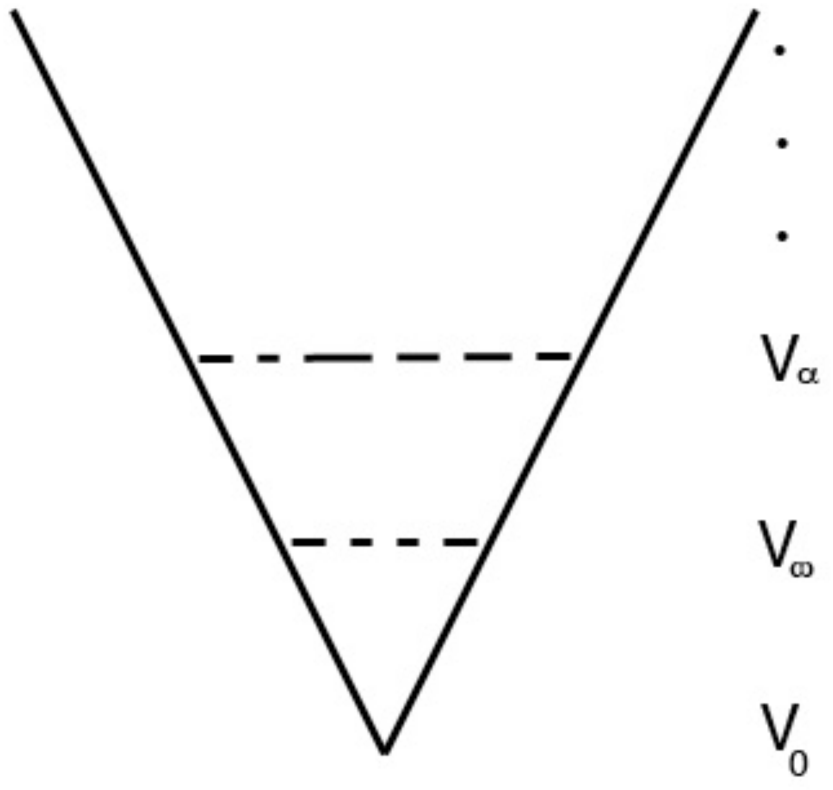
9. (*Axiom of Replacement*) Suppose X is a set and we replace each member x of X with some set y_x, according to some formula or well-defined rule. Then the resulting collection $\{y_x: x \in X\}$ is a set.

10. (*Axiom of Choice*) If X is a set whose members are nonempty sets, then there is a set Y which contains an element of each member of X.

The Universe of Sets

- Based mostly on Cantor's vision of the mathematical universe, an intuitive cumulative hierarchy V was envisioned that would be extensive enough to include all collections needed for mathematical practice but restrictive enough to avoid paradoxes
- This early version of V was used as a checkpoint as axioms were being formulated
- Once there was general agreement about the axioms, it was possible to build every stage of V in the formal theory – everything but the final leap to produce the totality V.

Every Mathematical Object Lives in the Universe of Sets: V



PART I : Background

2. Counting past 1, 2, 3, 4, . . .

Infinite Ordinals and Infinite Cardinals

- Sometimes it is necessary to count past the natural numbers. But how?
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- Examples of cardinals: $|\{ a,b,c \}| = 3$, $|\{0,1,2,3,\ldots\}| = \omega$

Panorama of Infinite Ordinals and Cardinals

ω_0	$\omega, \omega+1, \omega+2, \omega+3, \ldots, \omega+\omega, \omega+\omega+1, \omega+\omega+2, \ldots, \omega\cdot 3, \ldots, \omega\cdot\omega, \ldots, \omega^3, \ldots, \omega^\omega, \ldots$
ω_1	$\omega_1, \omega_1+1, \omega_1+2, \ldots, \omega_1+\omega, \ldots, \omega_1+\omega^2, \ldots, \omega_1+\omega^\omega, \ldots, \omega_1+\omega_1, \ldots, \omega\cdot\omega_1, \ldots, \omega_1^{\omega_1}, \ldots$
ω_2	$\omega_2, \omega_2+1, \omega_2+2, \ldots, \omega_2+\omega, \ldots, \omega_2+\omega^\omega, \ldots, \omega_2+\omega_1, \ldots, \omega_2+\omega_1^{\omega_1}, \ldots, \omega_2+\omega_2, \ldots, \omega_2^{\omega_2}, \ldots$
⋮	⋮
ω_α	$\omega_\alpha, \omega_\alpha+1, \omega_\alpha+2, \ldots, \omega_\alpha+\omega, \ldots, \omega_\alpha+\omega^\omega, \ldots, \omega_\alpha+\omega_1, \ldots, \omega_\alpha+\omega_2^{\omega_1}, \ldots, \omega_\alpha+\omega_\omega, \ldots, \omega_\alpha^{\omega_\alpha}, \ldots$
⋮	⋮

- Ordinals are used for counting; cardinals are used for naming sizes of sets
- FACT: Every set has a size. Either it has finite size (like 5 or 23) or infinite size (like ω_1 or ω_{23})

PART I : Background

3. *The Problem of Large Cardinals*
[Some infinite cardinals are too big…]

Large Cardinals

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- Regular strong limit cardinal = *inaccessible* cardinal
- Godel showed in 1936 that it is *impossible to prove inaccessibles exist or are even consistent* with ZFC
- Godel's Proof: If ZFC is consistent, ZFC cannot prove its own consistency. But if an inaccessible exists, ZFC is consistent. So ZFC can't prove such a cardinal exists!

Large Cardinals in Mathematics

- In the past 80 years, many large cardinals have been isolated.
- Examples of large cardinals (from weakest to strongest)

  inaccessible (Hausdorff)
  measurable
  strongly compact
  supercompact
  huge
  2-huge
  3-huge
  n-huge
  super-n-huge
  super-n-huge for every natural number n

Large Cardinals (continued)

- ***If they can't be proven to exist, why not just abolish large cardinals?***

Large cardinals show up as key elements in solutions to research problems (but, using ZFC alone, can't be proven to exist):

Sample Theorem: Lebesgue measure can be extended to a total measure if and only, in some universe, there is a measurable cardinal.

The Problem of Large Cardinals

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The Problem of Large Cardinals

- **The Problem**: Find one or more "natural" axioms that could be added to ZFC that would make it possible to *derive* the existence of large cardinals.
- **Intuition about the Infinite.** A deep intuition about the true nature of the infinite is required to arrive at such axioms.
- **Question** What is it about the nature of the wholeness V that would suggest that these large cardinals exist?

MVS Perspective

Maharishi Vedic Science Insights

- wholeness moves within itself without losing its essential nature
- its dynamics are unmanifest but present at every point in creation
- its dynamics involve a collapse to a point (from A to K) and expansion to infinity (from K to Veda to Vishwa to Brahm)

The Wholeness Axiom: An MVS Solution

- **The Wholeness Axiom (WA)** asserts: There is a nontrivial elementary embedding j : V -> V whose restriction to any set is itself a set.

  Fact: Such an embedding must be *undefinable* in the universe

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The Wholeness Axiom: An MVS Solution

- **The Wholeness Axiom (WA)** asserts: There is a nontrivial elementary embedding $j : V \to V$ whose restriction to any set is itself a set.

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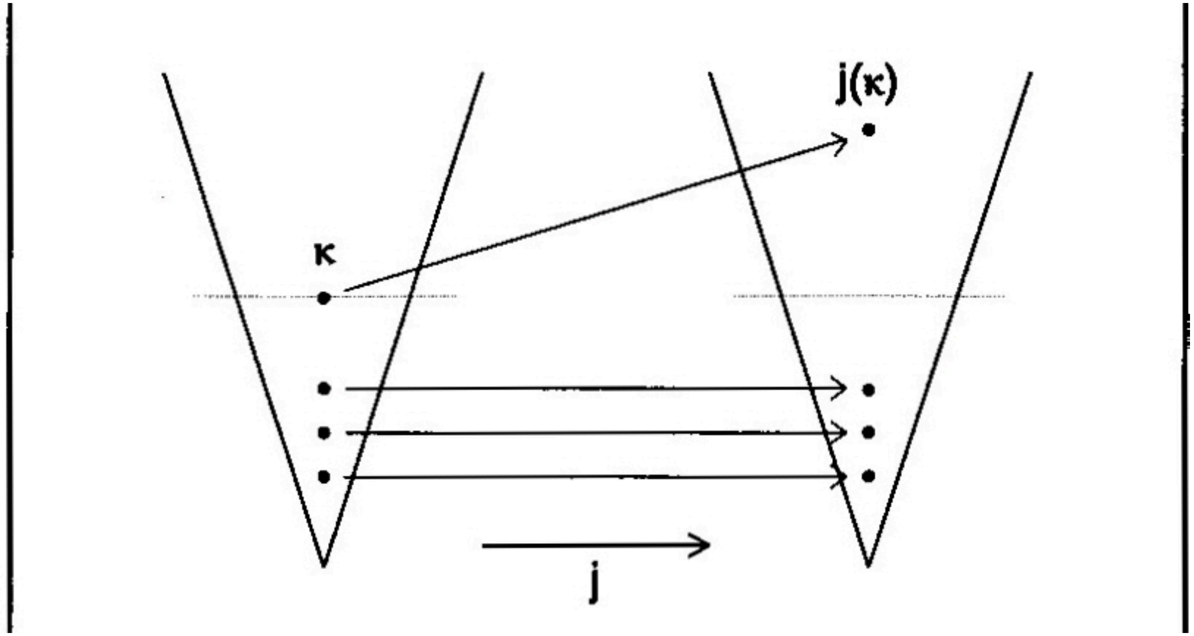
- **Theorem**. The least ordinal κ moved by j (the *critical point of j*) has (essentially) all the known large cardinal properties; in particular, κ is super-n-huge for every natural number n. Moreover, the κth stage of the universe is a full reflection of the totality V. κ can declare "I am the Totality".

A Solution to the Problem of Large Cardinals



WA As an Application of MVS

Maharishi Vedic Science	Set Theory
• Wholeness	• V, the universe of sets
• Move of wholeness within itself	• $j : V \rightarrow V$
• Wholeness unchanged by the transformation	• j is an *elementary* embedding
• Dynamics of wholeness present at every point in creation	• The restriction of j to any set is itself a (set) function in the universe
• Collapse of Infinity to a Point, imbued with infinite dynamism – collapse of A to K	• The critical point κ arises as the first point moved by j, imbued with the properties of wholeness of V, including all large cardinal properties
• Expansion of Point to Infinity – K expands to Veda to Vishva	• Interaction between j and κ expands to a Laver sequence which gives rise to all sets

PART I : Background

4. The need to revitalize the Axiom of Infinity

Axiom of Infinity in Modern Set Theory

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- Therefore, this axiom offers very little intuition about "what infinite objects exist" in the universe – no hint about whether or which large cardinals "really" exist in V
- The problem with the current Axiom of Infinity is summarized in quotes from Maharishi:

Creation arises from the natu-ral numbers 1, 2, 3, . . .

Viewing natural numbers as unconnected to their source is the beginning of ignorance

New Intuition about the Natural Numbers

- We ask: Where do the natural numbers come from? What is their *source?* What if we view 0,1,2,3,… as *precipitations of a field* rather than as being the reality of the infinite?
- Think of the infinite as the *dynamics of an unbounded field.* Think of the natural numbers as discrete crystallizations of these dynamics, each rooted in its underlying source of unboundedness
- This notion of infinity was suggested by R. Dedekind at the end of the 19^{th} century

Dedekind Self-Maps

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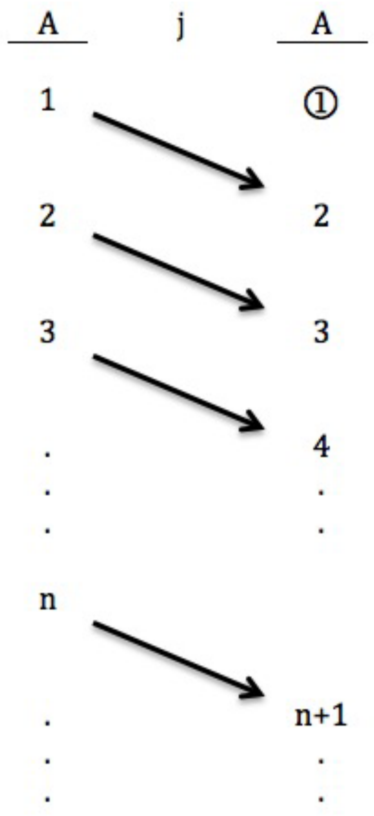
- Equivalent Idea: A Dedekind self-map is a function j: A -> A that is 1-1 but not onto; any point not in the range of f is called a critical point.

A Dedekind Self-Map with Critical Point = 1

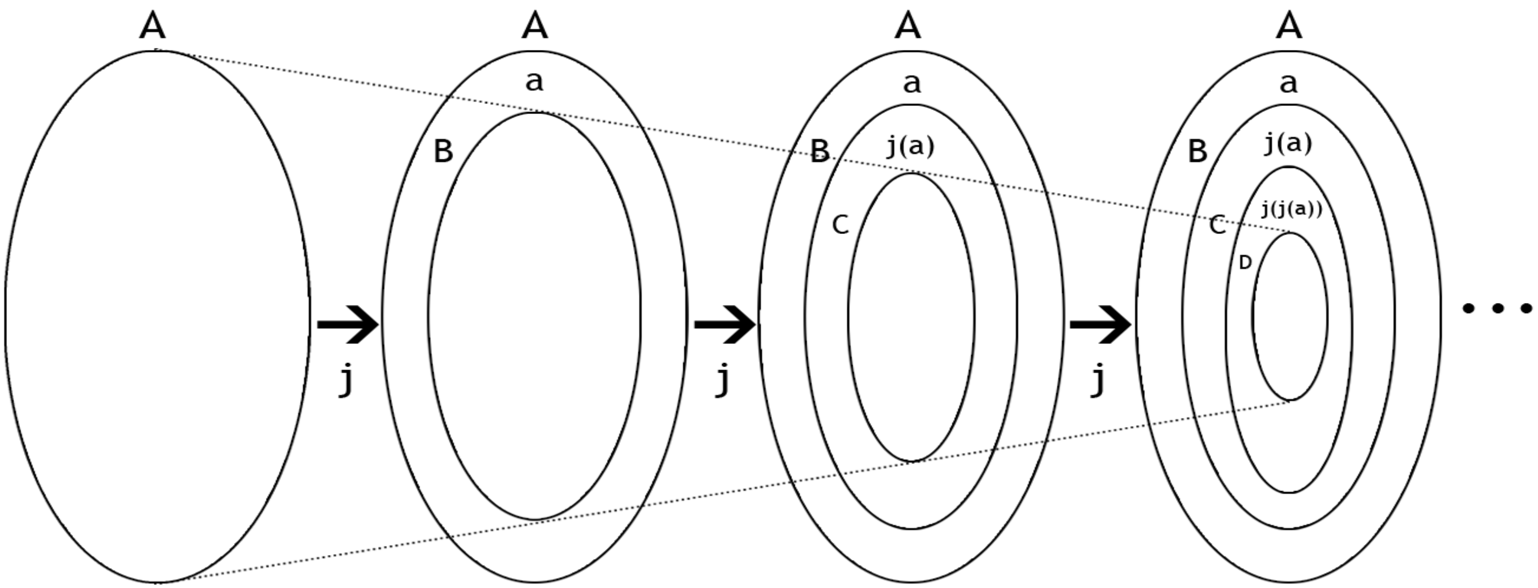


Dedekind Self-Map => Natural Numbers

- Given a Dedekind Self-Map j: A -> A with critical point a, one can repeatedly apply j to obtain the sequence

$$a, j(a), j(j(a)), j(j(j(a))), \ldots$$

Infinity of Transformations Within a Dedekind Self-Map j: A -> A



Within this flow by j, there emerges a sequence a, j(a), j(j(a)) . . . This is the *blueprint* of the natural numbers.

Dedekind Self-Map => Natural Numbers

- Given a Dedekind Self-Map j: A -> A with critical point a, one can repeatedly apply j to obtain the sequence

  $$a, j(a), j(j(a)), j(j(j(a))), \ldots$$

- One can *derive* from this sequence the actual natural numbers 0, 1, 2, . . ., and, more importantly, the actual successor function s (s(n) = n + 1).

Characteristics of a Dedekind Self Map

Consider a Dedekind self-map j: A -> A with critical point **a**.

- j *preserves* the essential character of A, that of being a *Dedekind infinite set* (j transforms A into B and B is Dedekind infinite)
- j *transforms A – j* is not just the identity function; values of A are moved by j
- j *has a critical point* **a** – the point **a** becomes a focal point for further self-transformation

Application to the Problem of Large Cardinals

The Problem of Large Cardinals is: How to account for the enormous cardinal numbers that appear in mathematics?

- First Try: Look to the (old) Axiom of Infinity to understand the "infinite" more clearly. Result: Not much.

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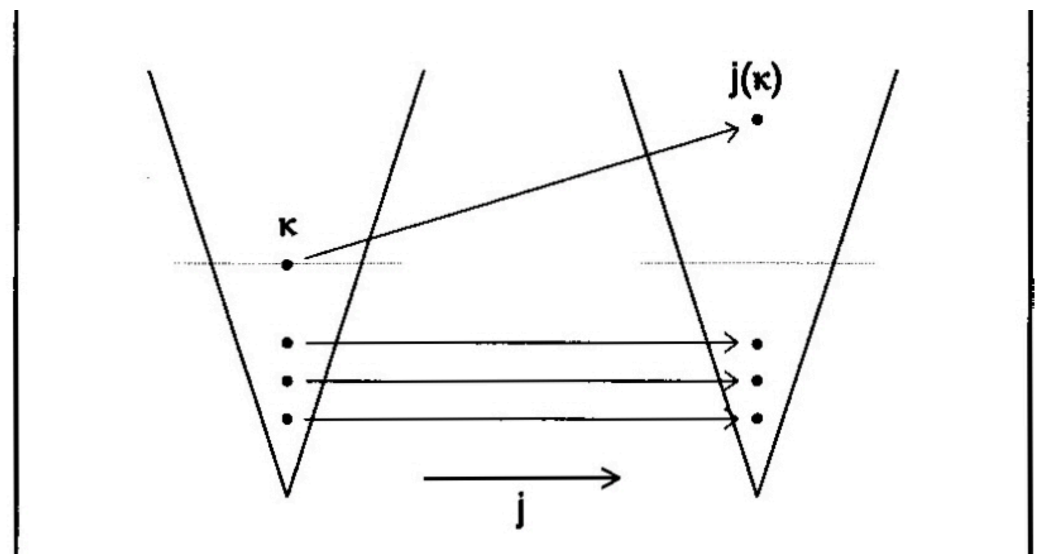
- Second Try: Use the new Axiom of Infinity ("there is a Dedekind self-map j: A -> A with critical point a") to get a hint about how to solve the problem
- Maybe large cardinals arise in the interaction of a Dedekind self-map *of the universe* with its critical point?

A Solution to the Problem of Large Cardinals



A solution is given by a Dedekind self-map j: V -> V with critical point κ (the least cardinal moved by j is a critical point of j), with the added feature that j is an *elementary embedding* whose restriction to any set is also a set.

PART II : Recent Progress

Question: Where do Dedekind self-maps come from?

Two reasons to ask:

1. Is there a *mathematical* motivation for introducing Dedekind self-maps?
   MVS gives us an intuition that the natural numbers should be viewed as arising from dynamics of an unbounded field. Is there a *mathematical reason* for adopting this view?

2.

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2. Dedekind self-maps of the universe are *too weak!*

   Even though a Dedekind self-map is equivalent to existence of an infinite set, a Dedekind self-map of the universe is not strong enough to imply existence of an infinite set. Do the dynamics of the wholeness of V – using just standard set theory – *suggest* a way to arrive at a Dedekind self-map? If we know how to arrive at the infinite from the finite, we may learn how to arrive at large cardinals from ordinary cardinals.

(continued)

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- *Observe* dynamics of the universe rather than *postulate*.

  Rather than just postulating that an infinite set, or a Dedekind self-map, exists, we try to detect how it might *emerge* in a natural way from dynamics already present in the universe.
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- For this study we work in the theory

  ZFC – Infinity

  In this world, infinite sets may or may not exist – and there is no way to prove either of these possibilities.

Inspiration from Algebraic Theories: How Algebraic Theories Emerge from Sets

Definition of a Group A *group* is a set G with an operation * satisfying :

- Associative Law: (g * h) * k = g * (h * k) for all g,h,k in G
- Identity: There is a special element e in G (the "identity") such that e * g = g = g * e for all g in G
- Inverses: Every g in G has an inverse – namely, an element g^{-1} with the property that $g * g^{-1} = e = g^{-1} * g$

Group Homomorphisms If G and H are groups, a function f: G -> H that preserves the operation is a *homomorphism*:

$$f(g_1 * g_2) = f(g_1) * f(g_2)$$

All Groups from Free Groups

- A certain type of group, called a *free group*, gives rise to all other groups by a collapsing operation.

- Every group G has a set X of generators:
  Example: Z (the group of integers under addition) is the free group with one generator – everything in Z can be generated from its element 1 using addition and additive inverse operation.

  1, 1+1, 1+1+1, …

  -1, (-1) + (-1), (-1) + (-1) + (-1) ,…

  0 = -1 + 1.

  In the example G = Z and X = {1}.

(continued)

- Given a group G with a generating set X, there is a free group F(X), generated *freely* by X, that gives rise to G by collapse

$$F(X) \rightarrow G \cong F(X)/H$$

where H is a particular subgroup of F(X).

(continued)

- In addition: Given any set X and a map g from X to some group G, there is a unique group homomorphism f: F(X)->G that agrees with g:

  f(x) = g(x) for all x in X

- This fact about free groups leads to a remarkable relationship between the world of sets and the world of groups

Emergence of Groups from Sets

- There is a natural 1-1 correspondence between functions from X to G (viewed in the world of sets) and group homomorphisms from F(X) to G (viewed in the world of groups)

- To distinguish between G as a set and G as a group, we introduce a map U that eliminates the group structure of G (by ignoring its operation) – so U(G) is just the underlying set of the group

- Example: Z is a group with addition whereas U(Z) is just the set { . . . -2, -1, 0, 1, 2 . . . }

Categories GRPand SET

- The collection of all sets together with maps between them is called the *category of sets* and is denoted SET
- The collection of all groups together with homomorphisms between them is called the *category of groups* and is denoted GRP
- For any two sets X,Y, SET(X,Y) denotes the set of all functions from X to Y
- For any two groups G, H, GRP(G,H) is the set of all group homomorphisms from G to H.

Adjoint Relationship Between F and U

- For any set X and any group G there is a 1-1 correspondence between GRP(F(X), G) and SET(X, U(G)), call it Θ

  Θ *:* GRP(F(X), G) -> SET(X, U(G))

- U represents a "silence" operation – it leads the structure of G back to its source as a set. F reprsents a "dynamism" operation – it creates a group F(X) "out of nothing" – just a bare set X.
- When these dynamic and silent operators are in balance, as they are here, they are said to form an *adjunction.*

F is left adjoint to U

GRP Emerges from SET via an Adjunction

- **Intuition**: Because the dynamic and silent influences that come from F and U are *balanced* – i.e. since F is left adjoint to G – free groups arise from sets and all groups arise from free groups.
- These dynamics are on the scale of the universe V – too large to be in the realm of sets. They provide a *context* for development of group theory from set theory. In a sense, these are *unmanifest dynamics.*
- Existence of a left adjoint F for a forgetful functor U
- **Question**: Can similar unmanifest dynamics cause something infinite to be born from the ZFC – Infinity world?

Category $V^{\circlearrowleft}$ of Self-Maps from V

- Make the internal self-referral dynamics of V – in the form of self-maps f: A -> A – explicit in a new category whose objects are themselves self-maps
- Think of V as another name for the category SET (objects are sets, as before)
- Write $V^{\circlearrowleft}$ for the category of self-maps (objects are self-maps)
- Since Dedekind self-maps are themselves self-maps from V, $V^{\circlearrowleft}$ is a natural "birthplace" for emergence of a Dedekind self-map

Emergence of $V^{\circlearrowleft}$ from V?

- It is natural to consider $V^{\circlearrowleft}$ as emerging from V through an adjunction, similar to the way GRP emerges from SET.
- Given a self-map f: X-> X, the "silence" operator ("forgetful functor") U is defined by:

$$U(f: X \to X) = X$$

(continued)

Does U have a left adjoint F?

- **Mathematical Intuition**: Slogan: "As many adjoints as possible exist" – there can be no proof that a left adjoint to U does *not* exist, the natural assumption is that it *should*
- **MVS Intuition**: The unfoldment of hidden dynamics of wholeness (like $V^{\circlearrowleft}$ from V) arises in the simultaneous flow of infinite dynamism and infinite silence – suggests a "yes" also

Consequences of the Adjunction

- Let $f_1 = F(\mathbf{1}) : X_1 \text{->} X_1$
- Let $\eta = \Theta_{1,f1}(\mathbf{1}_{f1})$ and let $\eta_o = \eta(o)$.
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Consequences of the Adjunction

- Let $f_1 = F(1) : X_1 \to X_1$
- Let $\eta = \Theta_{1,f_1}(1_{f_1})$ and let $\eta_0 = \eta(0)$.
- **Theorem.** $f_1 : X_1 \to X_1$ is a Dedekind self-map with critical point η_0.

What Happened?

- ***First Perspective:*** A Dedekind self-map arose from existence of a left adjoint for U. The balanced relationship between "dynamism" and "silence" in the interaction between V and its own self-maps is the ground for the emergence of the infinite in the world of ZFC – Infinity.
- ***Second Perspective*** Let j = G o F : V -> V. j is a Dedekind self-map of the universe with (least) critical point 1. And j(1) is a Dedekind-infinite set. From this perspective, the infinite arose from the dynamics of a very special Dedekind self-map of the universe.

Conclusion

- **MVS Offers a solution to The Problem of Large Cardinals**
  The problem demands a deep intuition about the structure of wholeness. Drawing upon the wisdom of MVS, we formulate a new axiom, the Wholeness Axiom, which essentially solves the problem – it shows that large cardinal properties are simply the powerful properties that are displayed in the first set moved by j : V -> V – like the properties inherent in K as A collapses to K

Conclusion (continued)

- **Meeting the Need for a More Informative Axiom of Infinity**. To address the problem of large cardinals on the basis of known mathematical intuitions, one looks to the Axiom of Infinity but discovers very little about the nature of the infinite.

  An MVS – inspired remedy is to view the natural numbers as precipitations of a *field*, rather than as being the final reality about the infinite. The "true" infinite is instead the dynamics of an unbounded field, captured in the notion of a Dedekind self-map. From a Dedekind self-map the natural numbers and successor function are derived, but such a map reveals much about the nature of the infinite, as a kind of flow.

Conclusion (continued)

- **Finding a Mathematical Starting Point for Dedekind Self-Maps or Any Kind of Infinity** Observing how algebraic categories emerge from the category of sets through definition of a left adjoint F to the forgetful functor U, it is natural to suppose that the same dynamics give rise to the category of self-maps $V^{\circlearrowleft}$. This intuition leads to the discovery of a Dedekind self-map emerging from the first point moved by F (namely, F(1)), whose Dedekind infinite domain X_1 arises from the critical point of a Dedekind self-map j: GoF: V->V of the universe .

Conclusion (continued)

- Unmanifest dynamics of the universe (adjoint relationship)
  => Strong Dedekind self-map j : V -> V (still unmanifest)
  => Dedekind self-map f: X -> X in realm of sets, producing an infinite set and a new Axiom of Infinity
  => Mathematical intuition for the notion of "Infinity" in mathematics, and hints about Problem of Large Cardinals
  => Stronger forms of Dedekind self-maps j: V->V, culminating in the Wholeness Axiom

MVS suggests the intuition regarding "what to look for" at each step of the solution.

Q & A: The Universe V As Wholeness - 1

- *all possibilities* All models of every mathematical theory are located in *V*. All sets needed for the development of any mathematical theory are located in *V*.

- *omniscience* Every mathematical fact is true in the model *V*. Thus, if one could view mathematics from the vantage point of *V*, the wholeness underlying mathematics, every mathematical truth could be known.

- *freedom* The power set axiom freely generates the set of all subsets of a given set. Since no restriction is placed on the sets generated in this way, the continuum may consistently be taken to have arbitrarily large cardinality.

- *unmanifest* *V* is too large to be an individual set; although all properties of sets can be rigorously determined and demonstrated using the axioms of set theory, nothing can be directly proven about *V*.

Q & A:
The Universe V As Wholeness - 2

- *simplicity* A single elegant recursive rule is at the basis of the sequential and simultaneous unfoldment of all stages of the universe.

- *omnipotence* Any mathematical truth that has ever been demonstrated can be seen as a derivation from the axioms of set theory using rules of logic, and all of these can be found in coded form within the structure of the universe itself.

- *total potential of natural law* The laws governing a mathematical theory are expressed by axioms. The content of every axiom of set theory is fully realized in the universe of sets.

- *discriminating* The sets which emerge in the cumulative construction of *V* do not lead to any known paradox.

Q & A:
The Universe V As Wholeness - 3

- *infinite silence* At limit stages of the construction of the universe, no new sets are added; this silent phase of the construction creates smoothness and uniformity in the universe.
- *infinite dynamism* In the construction of *V*, each new stage produced by the power set operator is larger than the previous stage; in particular, the power set operator produces an endless sequence of ever larger infinities.
- *pure knowledge* The information content in ZFC is the basis for essentially all known mathematical theorems.
- *infinite organizing power* The organizing power of a mathematical theory is expressed in its models. The models of set theory are infinite, complete, and all-inclusive.

Q & A: The Universe V As Wholeness - 4

- *perfect orderliness* All theorems of set theory, and hence of all of mathematics, can in principle be generated automatically by a computer once sufficiently many axioms have been input.
- *self-sufficiency* All the information needed to construct the stages of the universe is coded in the first few stages of the universe; the universe can therefore reproduce itself.
- *purifying* The recursive construction of V systematically prevents the entry of paradoxical sets.
- *infinite creativity* All the creativity of the brightest mathematicians of recorded history can be coded up as formal theorems derivable from the simple axioms of set theory.

Q & A:
The Universe V As Wholeness - 5

- *integrating* All mathematical theories, with their own special mathematical languages, find a common basis in set theory; the interrelationships between theories are thereby more easily identified.
- *harmonizing* Superficial differences in style between different theories are stripped away when the formal content of these theories is expressed in the language of set theory.
- *perfect balance* Despite the differences in style and content between different theories and their models, all such models naturally emerge in the uniform and simply defined unfoldment of the stages of the universe.
- *unboundedness* The sequence of stages of the universe *V* unfold without bound; the resulting universe *V* is so vast that it cannot be considered a set.
- *omnipresence* All mathematical structures can be located inside *V*.

What Is Missing in the ZFC Version of V?

- Knowledge of wholeness – of V as a whole
- Clear expression of self-referral dynamics.
- Collapse of unboundedness to a point in the emergence of sets

Elements of the Rosebrugh-Wood Adjoint String Theorem

For each category $\mathcal{C}$, we let $\mathcal{K}(\mathcal{C})$ denote $\mathbf{Set}^{\mathcal{C}^{op}}$.[58] $\mathcal{K}$ is a morphism (in fact, a contravariant functor) in **Cat**, so it is defined on morphisms of **Cat** in the following way: Suppose $\mathcal{C}, \mathcal{D}$ are categories and $\mathbf{F} : \mathcal{C} \to \mathcal{D}$ is a **Cat**-morphism. Then $\mathcal{K}(\mathbf{F}) : \mathbf{Set}^{\mathcal{D}^{op}} \to \mathbf{Set}^{\mathcal{C}^{op}}$ is defined by $\mathcal{K}(\mathbf{F})(\mathbf{H}) = \mathbf{H} \circ \mathbf{F}$.

Lemma R_1. If $\mathbf{L}_1, \mathbf{L}_2$ are left adjoint to a functor $\mathbf{F}$, then $\mathbf{L}_1$ and $\mathbf{L}_2$ are naturally isomorphic. Likewise, if $\mathbf{R}_1, \mathbf{R}_2$ are both right adjoints of $\mathbf{F}$, then $\mathbf{R}_1$ and $\mathbf{R}_2$ are naturally isomorphic. ■

Lemma R_2. Whenever $\mathbf{L} \dashv \mathbf{F}$, we have $\mathcal{K}(\mathbf{L}) \dashv \mathcal{K}(\mathbf{R})$. ■

Lemma R_3. If the morphisms of $\mathcal{C}$ form a *set* and if $\mathcal{D}$ is locally small, and $\mathbf{F} : \mathcal{C} \to \mathcal{D}$, then $\mathcal{K}(\mathbf{F})$ has a left adjoint, denoted $\exists\mathbf{F}$ and a right adjoint, denoted $\forall\mathbf{F}$. Therefore,

$$\exists\mathbf{F} \dashv \mathcal{K}(\mathbf{F}) \dashv \forall\mathbf{F}.$$

In particular, if $\mathbf{F}$ is the Yoneda embedding $\mathbf{Y}_{\mathcal{C}} : \mathcal{C} \to \mathcal{K}(\mathcal{C}) = \mathbf{Set}^{\mathcal{C}^{op}}$, then $\forall\mathbf{Y}_{\mathcal{C}} \cong \mathbf{Y}_{\mathcal{K}(\mathcal{C})}$. Therefore:

$$\exists\mathbf{Y}_{\mathcal{C}} \dashv \mathcal{K}(\mathbf{Y}_{\mathcal{C}}) \dashv \mathbf{Y}_{\mathcal{K}(\mathcal{C})}. \blacksquare$$