"Indestructibility of Wholeness"

Fundamenta Mathemticae, 2020

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Know That to be indeed indestructible by which all this is pervaded. None can work the destruction of this immutable Being.

-- Maharishi, *Gita*, verse II.17

Research Program: Enliven wholeness in the foundation of mathematics to align the mathematical enterprise more fully with Natural Law

Publication Timeline (high points)

2000 – *The Wholeness Axiom and Laver Sequences*, Ann Pure and App Logic

2006 – *The Spectrum of Elementary Embeddings $j: V \to V$*, Ann Pure and App Logic

2010 – *The Axiom of Infinity and Transformations $j: V \to V$*, Bull Symb Logic

2016 – *Magical Origin of the Natural Numbers*, IJMAC, Vol. 2 (to appear in a mathematics journal in a more technical form)

2020 - *Indestructibility of Wholeness*, Fundamenta Mathematicae

Overview of the Research

1. **Emergence of Exotic Infinities: "Large Cardinals"**
   - Historically, the study of the Mathematical Infinite led to the discovery of exotic infinite sets (known as *large cardinals*) that could not be accounted for on the basis of the known axioms at the basis of mathematics
2. **Accounting for Large Cardinals with the Wholeness Axiom**
   - By adding a new axiom to the known axioms – an axiom called the *Wholeness Axiom,* which talks about the *nature of the Wholeness of the mathematical universe* – it was discovered that nearly all these exotic infinities could be understood and derived in a straightforward way.
3. **Creation of new universes cannot destroy the Wholeness Axiom – Wholeness is *indestructible***
   - One test to see if the Wholeness Axiom is a *good axiom* is to see whether it survives the transformational dynamics that arise from *creating new universes*. The paper "Indestructibility of Wholeness" shows that once the Wholeness Axiom is seen to be true in one universe, it will continue to hold true in any new universe that is created from the starting universe using the method of *forcing*.

Exotic Infinities (Large Cardinals)

Begin with this question: Compare $\mathbb{N}$ and $\mathbb{R}$. Which set is bigger, or do they have the same size?

$$\mathbb{N} = \{1, 2, 3, 4, 5, 6, 7, 8, \ . \ . \ . \}$$

$\mathbb{R}$ ________________________________

Cantor's Theorem. *There are more real numbers than there are natural numbers.*

$$\mathbb{N} < \mathbb{R}$$

Georg Cantor was a 19th century mathematician who revolutionized the mathematical understanding of the infinite

Exotic Infinities (Large Cardinals)

Cantor showed that $\mathbb{N}$ is a smaller infinity than $\mathbb{R}$. Is there anything bigger than $\mathbb{R}$?

Subsets of a Set. For any set X we can collect together all *subsets* of X to form a new set P(X).

Example:

X = {1, 3}

subsets of X: {1}, {3}, {1,3}, {}

P(X) = {{1}, {3}, {1,3}, {}}

Note!
P(X) is a bigger set than X!

Cantor's Theorem 2. *There are more subsets of* $\mathbb{R}$ *than there are points in* $\mathbb{R}$. *In fact, for any set X, X < P(X).*

Exotic Infinities (Large Cardinals)

Cantor's work showed that there is an ENDLESS hierarchy of every larger infinities in mathematics

$$N < P(N) < P(P(N)) < P(P(P(N))) < \quad . \quad . \quad .$$

Exotic Infinities (Large Cardinals)

Cantor invented symbols to represent every possible infinite size. "Sizes" of sets are called CARDINALS.

The *finite* cardinals are well-known to everyone:

0, 1, 2, 3, 4, . . .

The size of the set { a, b, c, d } is represented by the cardinal number 4.

The size of the set N of natural numbers is denoted ω or ω_0 (pronounced "omega" or "omega-zero")

Cantor's List of Infinite Cardinals

$\omega_0 \quad \omega_1 \quad \omega_2 \quad \omega_3 \quad \omega_4 \quad . \; . \; .$

What comes next?

$\omega_\omega \quad \omega_{\omega+1} \quad \omega_{\omega+2} \; . \; . \; . \; \omega_{\omega+\omega} \quad . \; . \; .$

$\omega_{\omega_1} \quad \omega_{\omega_2} \quad \omega_{\omega_3} \quad \ldots \quad \omega_{\omega_\omega} \quad \ldots$

Exotic Infinities (Large Cardinals)

Every conceivable infinite size (or cardinal) must lie in Cantor's list of infinite cardinals. Which ones in Cantor's list are LARGE CARDINALS?

It's IMPOSSIBLE to precisely locate a large cardinal in the list. It is possible illustrate one of the properties large cardinals must have (could be done during Q&A).

Exotic Infinities (Large Cardinals)

A List of Popular Large Cardinals

Inaccessible cardinal

Weakly compact cardinal

Measurable cardinal

Woodin cardinal

Supercompact cardinal

Huge cardinal

Superhuge cardinal

Super-*n*-huge for every *n* cardinal

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1. Emergence of Exotic Infinities: "Large Cardinals"
2. Accounting for Large Cardinals with the Wholeness Axiom
3. Creation of new universes cannot destroy the Wholeness Axiom – Wholeness is *indestructible*

Accounting for Large Cardinals with the Wholeness Axiom

To understand the Wholeness Axiom, we need to understand the concept of an *AXIOM*.

About *Axioms*

- Axioms are first principles that are taken to be fundamental truths.
- All mathematical objects can be represented as sets, so the fundamental axioms for mathematics are axioms about sets.
- The axioms at the basis for all of mathematics are known collectively as the *Axioms of ZFC*
- The ZFC axioms tell us the fundamental properties that sets must have and provide an instruction manual for building a universe of sets

Accounting for Large Cardinals with the Wholeness Axiom

Examples of ZFC Axioms

Pairing Axiom. If X and Y are sets, there is another set Z that has X and Y as its only elements. (*Notation*: Z={X, Y})

Power Set Axiom. If X is a set, the collection P of all subsets of X is also a set. (*Notation*: P = P(X))

Axiom of Infinity. There is an infinite set. (*Equivalently*: The natural numbers 1, 2, 3, . . . can be collected together to form a set.)

Accounting for Large Cardinals with the Wholeness Axiom

Using the ZFC Axioms to build the universe V of sets

The ZFC Axioms tell us how to build the universe *V* by piecing together a vast collection of ever larger *partial* universes V_0, V_1, V_2, . . .

$$V_0 = \emptyset$$

(V_0 is the *empty set*)

$$V_1 = \mathcal{P}(V_0) = \{\emptyset\}$$

$$V_2 = \mathcal{P}(V_1) = \{\emptyset, \{\emptyset\}\}$$

$$V_3 = \mathcal{P}(V_2) = \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\}$$

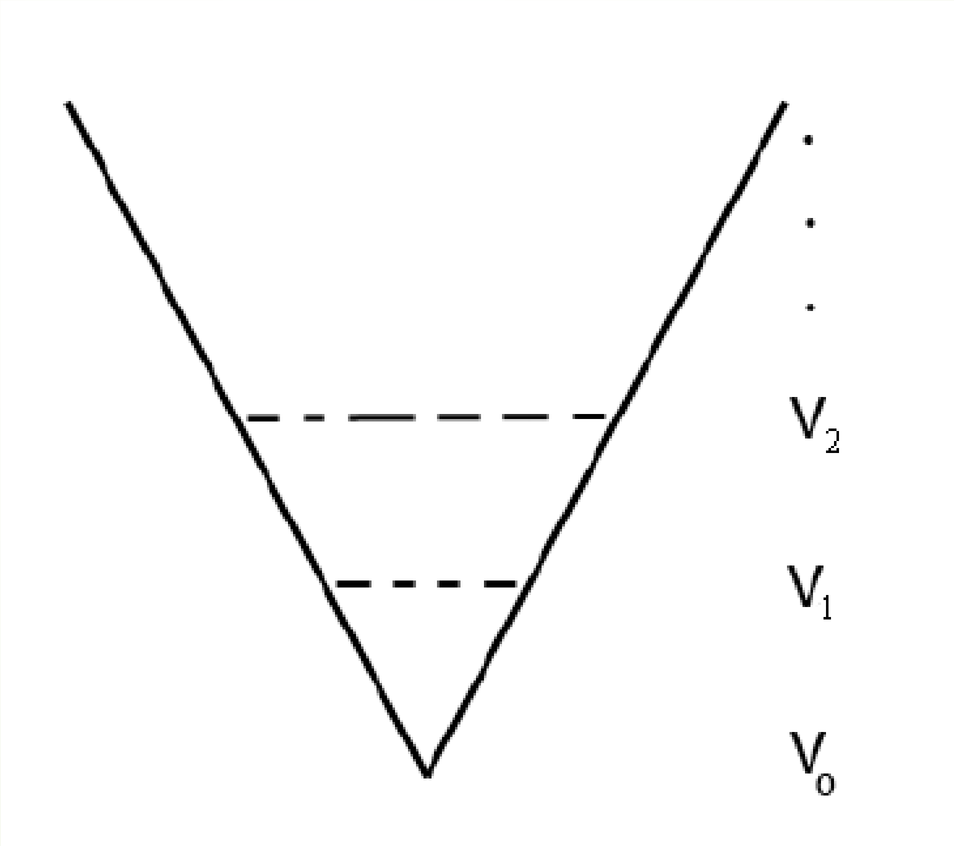
$$\cdot = \cdot$$

$$\cdot = \cdot$$

$$\cdot = \cdot$$

Accounting for Large Cardinals with the Wholeness Axiom

$$V = V_0 \cup V_1 \cup V_2 \cup V_3 \cup \cdots$$



Accounting for Large Cardinals with the Wholeness Axiom

Fact 1. The ZFC axioms provide a foundation for 99.99% of mathematics.

Fact 2. The ZFC axioms CANNOT prove the existence of large cardinals.

Conclusion. A new axiom must be added to the current list of ZFC axioms in order to account for large cardinals. This step requires a deep insight into the nature of the Mathematical Infinite.

Big Clue. The ZFC axioms do not talk at all about the nature of the *wholeness* of the universe; they talk only about the parts.

Accounting for Large Cardinals with the Wholeness Axiom

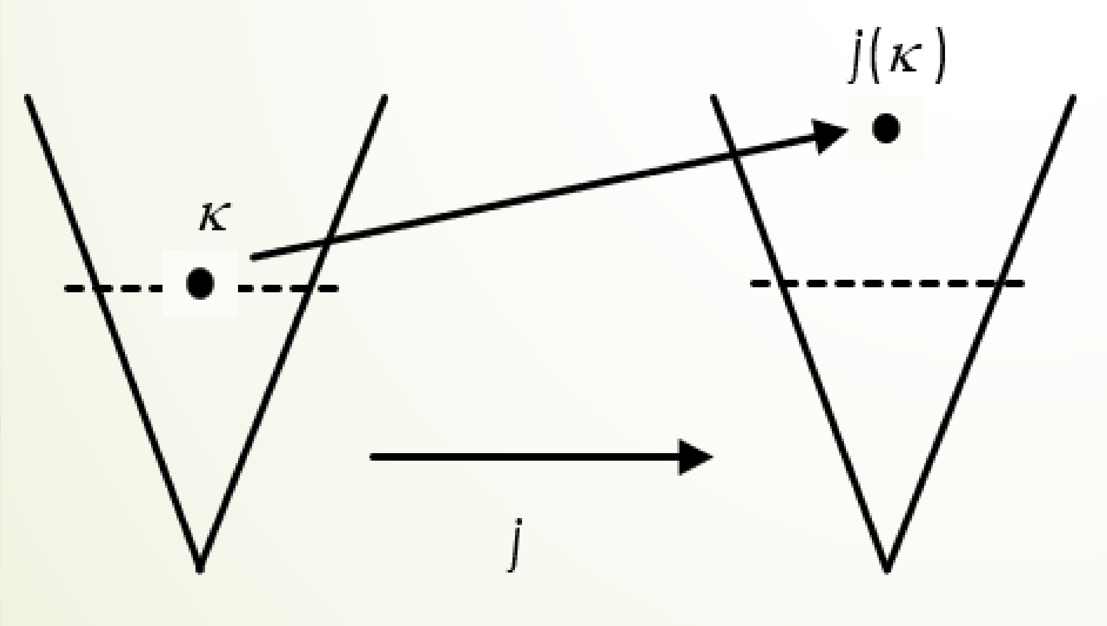
A Solution. Introduce an axiom that gives expression to the wisdom of Maharishi Vedic Science concerning wholeness:

The nature of wholeness is to transform itself within itself remaining unchanged by its own transformations.

"Pure consciousness, pure Being, is maintained always as pure consciousness and pure Being, and yet It is transformed into all the different forms and phenomena. Here is the cosmic law: one law which never allows absolute Being to change. Absolute Being remains absolute Being throughout, although it is found in changed forms at every level." *Science of Being*, p. 9.

Accounting for Large Cardinals with the Wholeness Axiom

The Wholeness Axiom. There is an elementary embedding $j : V \to V$ for which there is a smallest cardinal κ moved (that is, $j(\kappa) \neq \kappa$).



- V represents wholeness
- $j : V \to V$ represents the transformational dynamics of wholeness
- κ is the "point" – representing the first sprouting of activity of j
- Interaction between j and κ generates a "blueprint" (a *Laver sequence*)
- Interaction between j and the blueprint generates every object in the universe
- κ has all large cardinal properties, through super-n-huge for every n

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Wholeness Is Indestructible

Typically, mathematicians are involved in the following two activities:

1. Proving certain mathematical statements are true
2. Proving certain mathematical statements are false

Experts in Foundations of Mathematics (in the past 60 years) are involved also in a third activity:

3. Proving that a mathematical statement can neither be proved nor disproved

This third activity is accomplished by creating alternative universes of sets using the technique of **Forcing**

Wholeness Is Indestructible

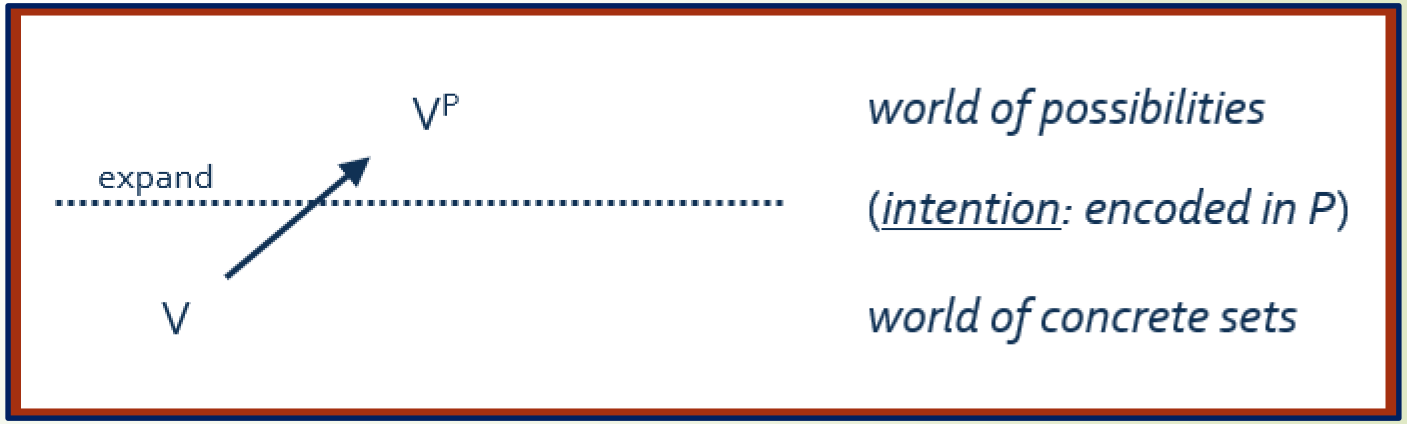
The Technique of Forcing (Creating New Universes of Sets)

1. Start with a statement S that you hope to prove is neither provable nor unprovable. You will want one universe in which S is true, another in which S is false. Consider the first case – try to make S true.

2. Come up with a set P (called a *notion of forcing*) that expresses your intention to make S true

3. Expand the universe V, using P, to an *all possibilities state* V^P.

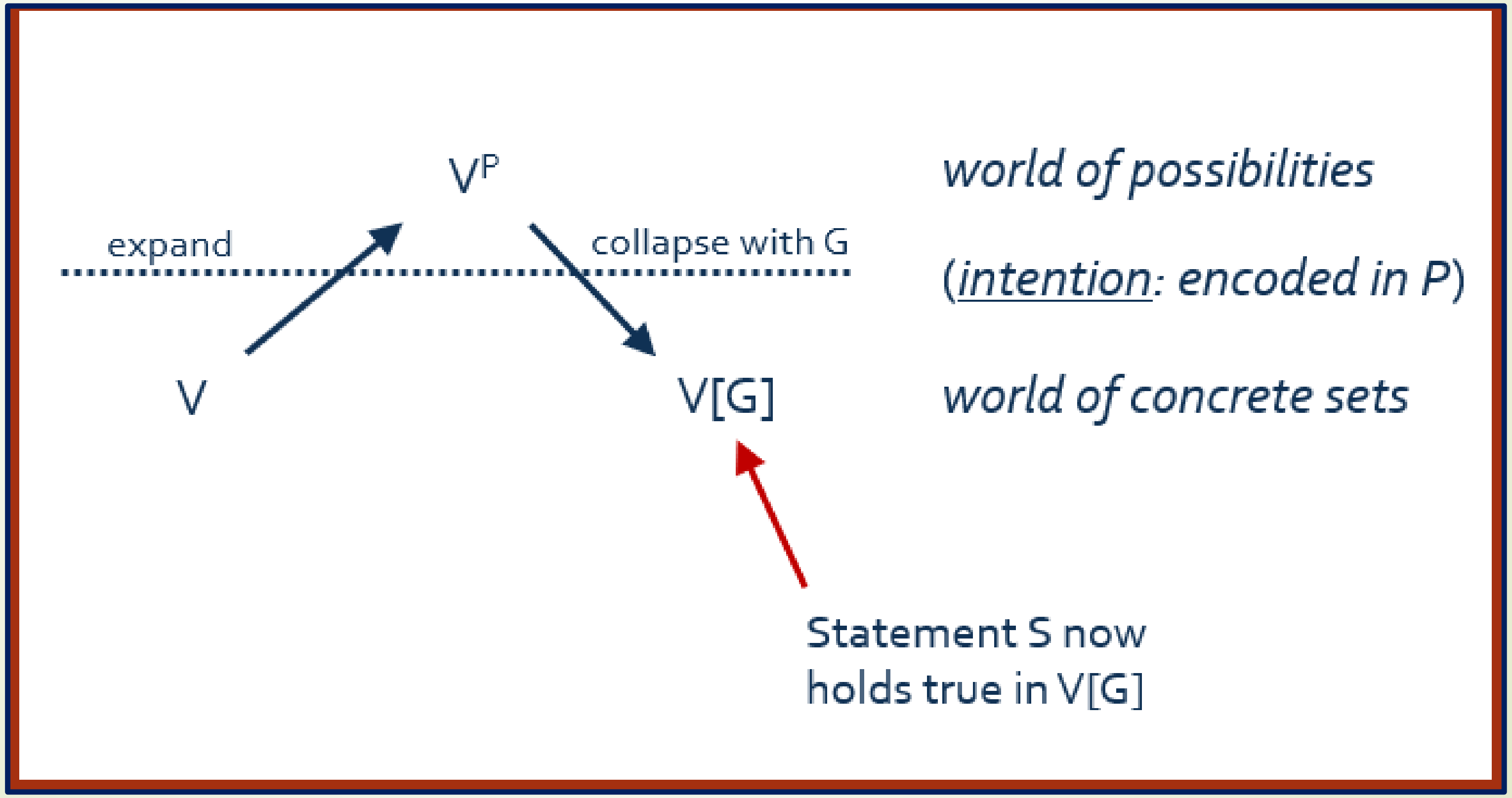
   Elements of V^P are no longer sets, but are *potential* sets.



Wholeness Is Indestructible

4. *Collapse* V^P using an "ideal set" G.

G is not a set in the universe V, but techniques in logic allow us to assume its existence



Wholeness Is Indestructible

Classical Example: Deciding the Continuum Hypothesis

1. Cantor showed N < R
2. Which cardinal in Cantor's list specifies the exact size of R?

$$\omega_0 \quad \omega_1 \quad \omega_2 \quad \omega_3 \quad \omega_4 \; . \; . \; .$$

3. Continuum Hypothesis (CH):

$$\text{size of R} = \omega_1$$

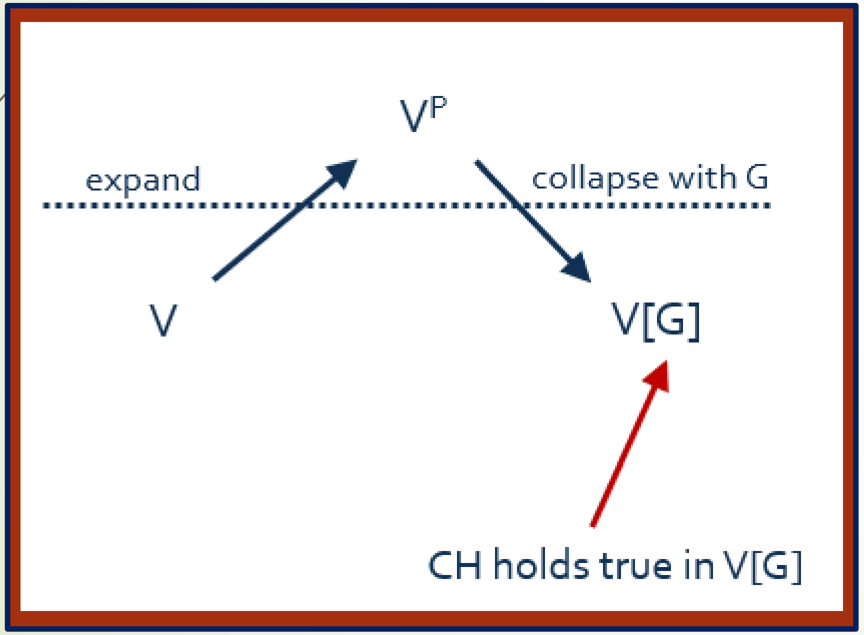
   Is CH true?

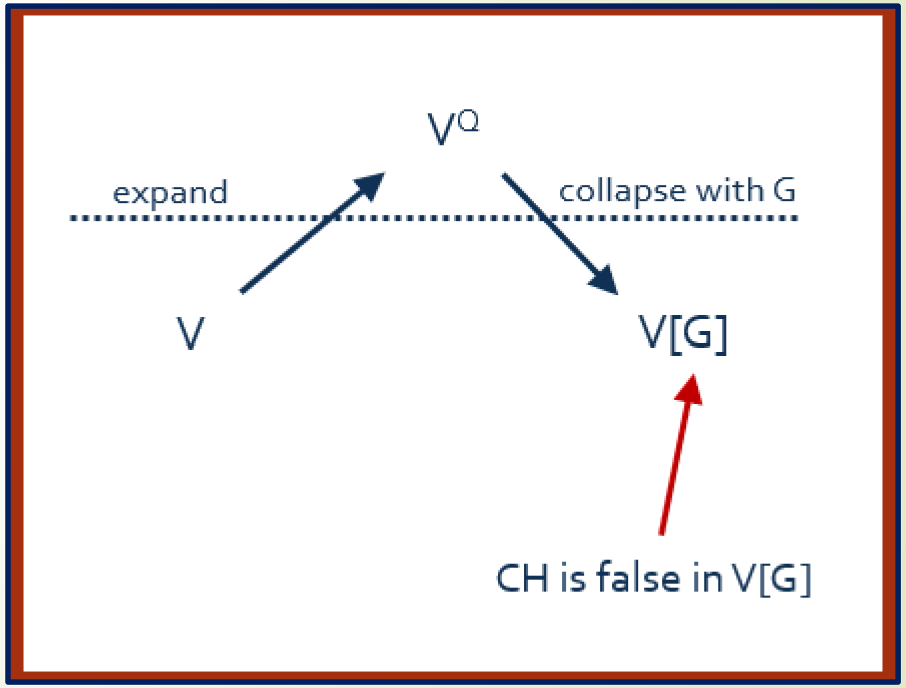
4. Determining the truth of the Continuum Hypothesis was considered by David Hilbert (1900) to be the #1 unsolved problem in mathematics.

Wholeness Is Indestructible

Settling the Continuum Hypothesis

Researchers discovered sets P and Q so that forcing with P makes the Continuum Hypthesis true and forcing with Q makes the Continuum Hypothesis false





Wholeness Is Indestructible

What happens to the Wholeness Axiom after forcing? If the Wholeness Axiom is accepted as a true axiom and holds in V, does it still hold in V[G] (after forcing)?

- After forcing, all the axioms of ZFC continue to hold true in V[G].
- A criterion for a new axiom of set theory to be acceptable is that it is preserved by forcing – forcing should not destroy the truth of the axiom.

Wholeness Is Indestructible

Main Result of New Publication:

Theorem. If the Wholeness Axiom holds in V, then for any notion P of forcing, the universe V[G] obtained from forcing with P also satisfies the Wholeness Axiom. Wholeness is indestructible.

Q & A

The Fixed Point Property of Large Cardinals

Notice:

0 is smaller than ω_0

1 is smaller than ω_1

2 is smaller than ω_2

3 is smaller than ω_3

$\vdots$

ω is smaller than ω_ω

$\vdots$

It looks like the *INDEX* of an infinite cardinal is always smaller than the CARDINAL.

Could the INDEX ever EQUAL the CARDINAL? Such a cardinal is called a FIXED POINT, and it is possible to build one.

$$\omega, \omega_\omega, \omega_{\omega_\omega}, \omega_{\omega_{\omega_\omega}}, \ldots \longrightarrow \omega_{\omega_{\omega_{\ldots}}}$$

IF

$$\alpha = \omega_{\omega_{\omega_{\ldots}}}$$

THEN

$$\alpha = \omega_\alpha$$