



# The Mathematics of Pure Consciousness

*Paul Corazza, Ph.D.*

# Pure Consciousness Is All There Is

*"Brahman alone is real, the world is mithya (not independently existent), and the individual self is nondifferent from Brahman."*

-- Shankara, Crest-Jewel of Discrimination

# Natural Question

Q. How do we account for the apparent diversity of the universe?

# The Insight of Vedanta and Maharishi Vedic Science

1. Being consciousness, pure consciousness is conscious of itself – so assumes role of both knower and known
2. The process of knowing itself is also the act of being conscious – so pure consciousness is also the process of knowing.
3. Diversity therefore arises within unity (Samhita):
  - + Rishi – knower
  - + Chhandas – known
  - + Devata – process of knowing

# First Impulse

Master Vasistha adds:

*My son, when, in the infinite consciousness, the consciousness becomes aware of itself as its own object, there is the seed of ideation.*

-- Yoga Vasistha, p. 190

# Maharishi's Elaboration

First syllable of Rk Veda is AK. A – infinite unboundedness -- collapses to K, a point value. Pure consciousness sees itself as a “point”

- A = infinite unbounded value of pure consciousness
- K = a point within pure consciousness

From this primordial collapse emerge all the transformational dynamics of pure consciousness within itself, as Veda

*Richo akshare parama vyoman*

The hymns of the Veda emerge in the collapse of 'A', the 'kshara' of 'A'.  
-- Rig Veda 1.164.39

# The Appearance of Diversity Due to *Vivarta*

Maharishi:

- *Here, Unity (Samhita) appears to be diversity (Rishi, Devata, and Chhandas). This is the absolute eternal principle of vivarta, where something appears as something else. The very structure of knowledge (Samhita) has the principle of vivarta (Rishi, Devata, Chhandas) within it*

-- Absolute Theory of Defense p. 589

- *The principle of vivarta makes the unmanifest quality of self-referral consciousness appear as the Veda and Vedic Literature, and makes the Veda and Vedic Literature appear as Vishwa*

-- pp. 377, 589.

# What Accounts for Our Experience?

- What we see is nothing but the dynamics of pure consciousness. Our experience of that reality depends on the level of consciousness.

*[In unity consciousness] the boundaries do not disappear . . . only they cease to dominate. Where before they were opaque . . . they are now fully transparent*

-- Maharishi, Conversations p. 47

- In Ignorance, diversity entails separation of parts from their source – this is pragya-aparadh:

*Pragya aparadh results when, in the mechanics of creation from the field of consciousness, the intellect loses sight of the essential unity which is the true nature of the self . . . The intellect gets caught up in its own creation, i.e., gets overshadowed by the perception of diversity to the exclusion of the unity which is the actual nature of the self being discriminated. According to Maharishi, this mistake of the intellect is so fundamental to the nature of human experience that it is responsible for all problems and suffering in life.*

--Hagelin, Restructuring Physics,



# Can We Model These Dynamics?

- Is there something in the mathematical universe that is a faithful analogue to pure consciousness, with all its internal dynamics?
- + Finding a mathematical model could lead to a deeper insight about the ultimate reality

# First Try: The Empty Set

- The mathematical universe is an analogue to the diversity of *our* universe.
- Hypothesis:  
*The empty set is an analogue to pure consciousness*

# The Mathematical Universe

## Three things to know:

1. Everything in mathematics is a set.  $\{1, 3, 7\}$  is a set, but so are
  - the ordered pair  $(2,3)$
  - the function  $f(x) = x^2$
  - the operations of addition and multiplication
  - logical connectives 'and' and 'or'
  - every kind of Hilbert space
  - every multi-dimensional space that people talk about.

*Important:* The whole numbers are defined as sets in a special way

$$0 = \emptyset \text{ (the empty set)} \quad 1 = \{0\}, \quad 2 = \{0, 1\}, \quad 3 = \{0, 1, 2\} \dots$$

# Three things - continued

2. Axioms of Set Theory. The Axioms tell us that certain sets exist and give us the rules that govern creation of sets from other sets.

Examples of some of the Axioms:

- *Axiom of Empty Set*. There is a set with no element.
- *Axiom of Pairing*. For any sets  $X, Y$  there is a set whose only elements are  $X, Y$  (denoted  $\{X, Y\}$ )
- *Power Set Axiom*. For any set  $X$  there is a set, denoted  $P(X)$ , whose elements are precisely the subsets of  $X$ .

# Three things - continued

## The construction of the universe $V$

- $V$  is built in stages  $V_0, V_1, V_2, \dots$

$V_0 = \emptyset$  (the empty set)

$V_1 = \{\emptyset\} = P(V_0)$

$V_2 = \{\emptyset, \{\emptyset\}\} = P(V_1)$

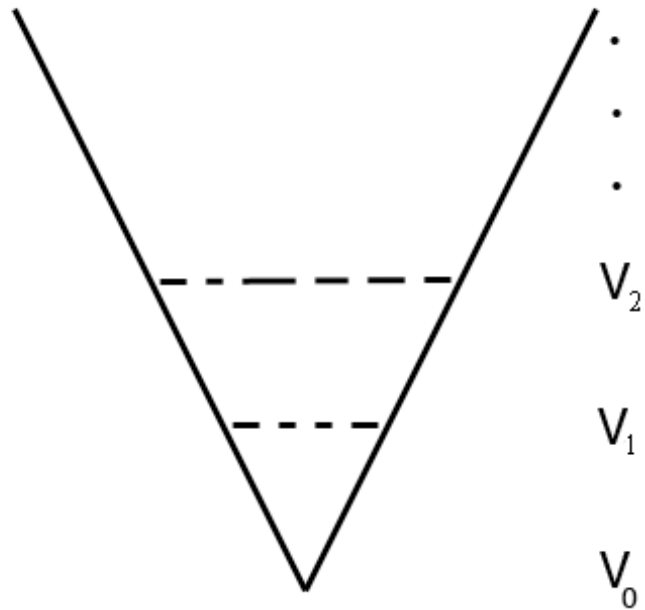
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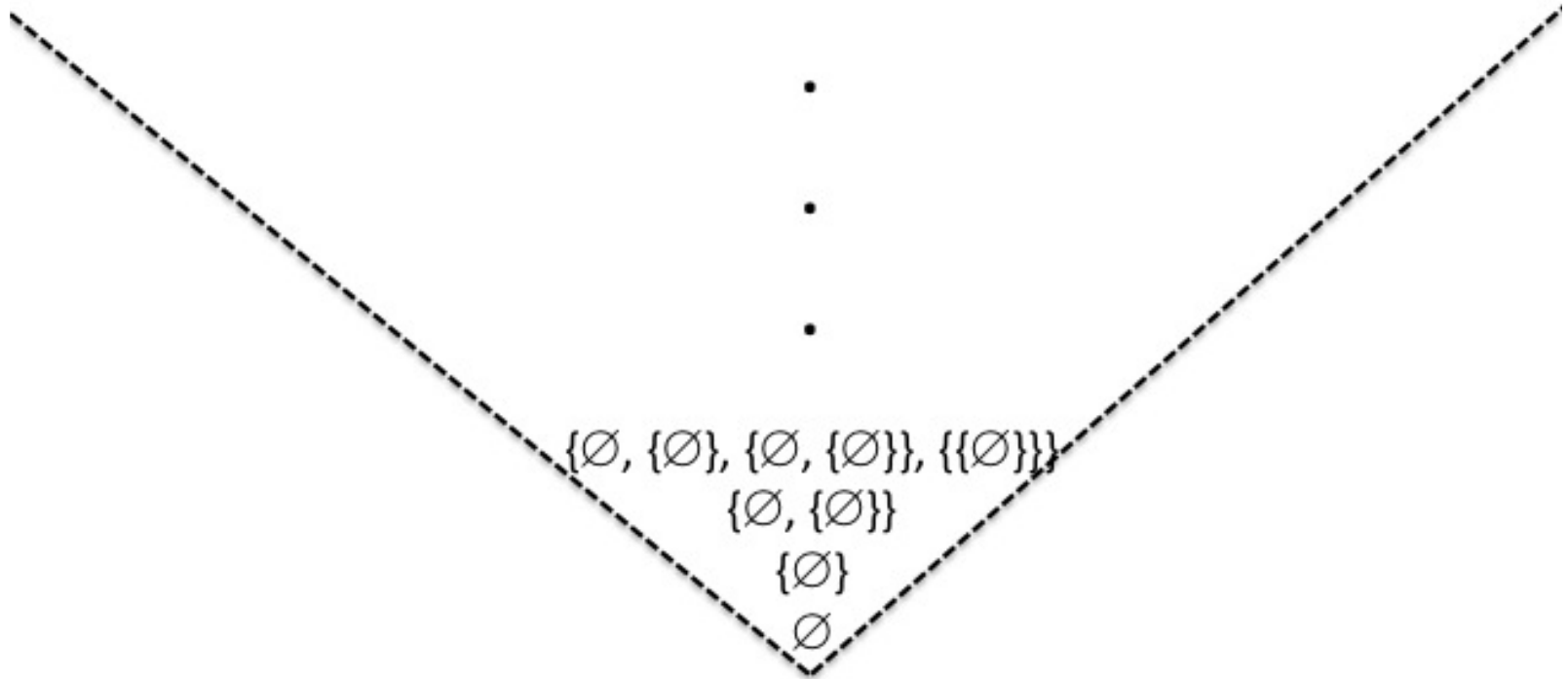
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- $V$  is obtained by forming the union of all the stages

# The Universe $V$



# Sets Are Different Views of the Empty Set



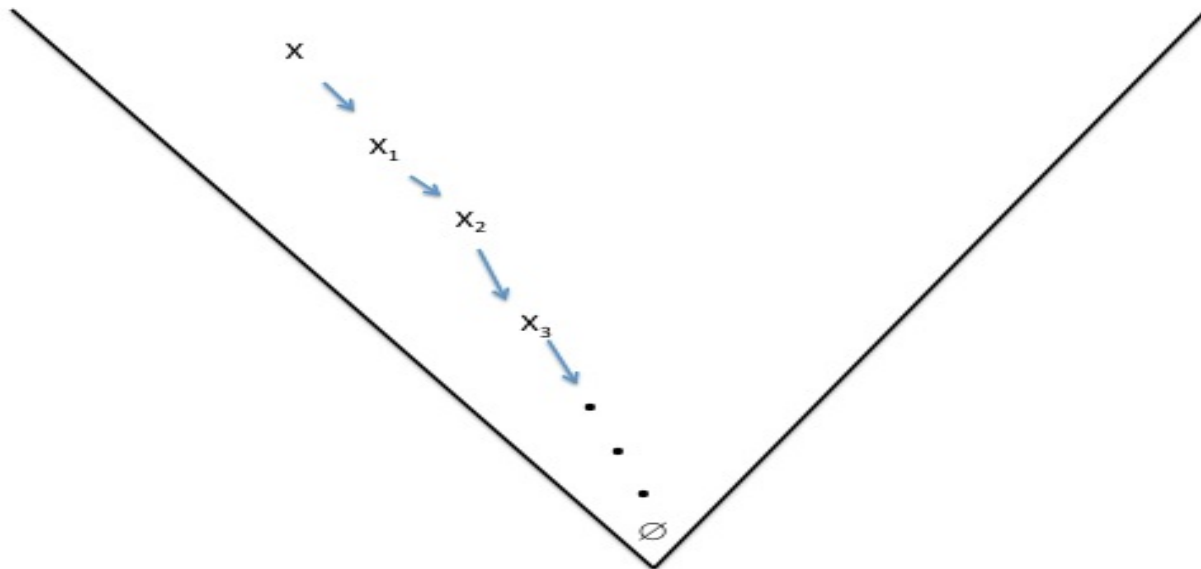
# Empty Set and the Singularity

- Empty set has no contents – it is unadulterated, pure
- Every set – every mathematical object – is nothing other than different ways of arranging empty sets and sets of empty sets and sets of sets of empty sets ...
- Every set can locate its source in the empty set through finitely many “steps of transcending”
- Notation:  $a \in B$  means “a is an element of the set B”



# Transcending to the Empty Set

Start with a set  $X$ . Find an " $\in$ -minimal" element  $X_1$  of  $X$ . Find an  $\in$ -minimal element  $X_2$  of  $X_1$ . Keep going. Since there are no infinite descending  $\in$ -chains in  $V$ , you must arrive at  $\emptyset$  after finitely many steps.



# Something Missing . . .

- In the “dynamics” of  $\emptyset$ , in the emergence of  $V_1$  from  $V_0$ , there is a “collapse” of  $\emptyset$  to  $\{\emptyset\}$  --  $\emptyset$  is seen as an “object”.
- But . . .
  - The dynamics by which  $\{\emptyset\}$  emerges from  $\emptyset$  are *not contained within*  $\emptyset$
  - Worse: Rather than maintaining unity,  $\{\emptyset\}$  is *separate from (unequal to)*  $\emptyset$ . This is the beginning of *pragya-paradh!*
  - This separateness propagates through the universe. Unity is lost.

# Loss of Connectedness Leads to Problems

*Viewing the natural numbers as unconnected to their source is the beginning of ignorance.*

-- Maharishi (1994)

# New Approach: Try to Solve the Equation

$$x = \{x\}$$

- + No solution can be found in  $V$ . Reason: If there were such a set  $x$ , we would have an infinite descending  $\in$ -chain.

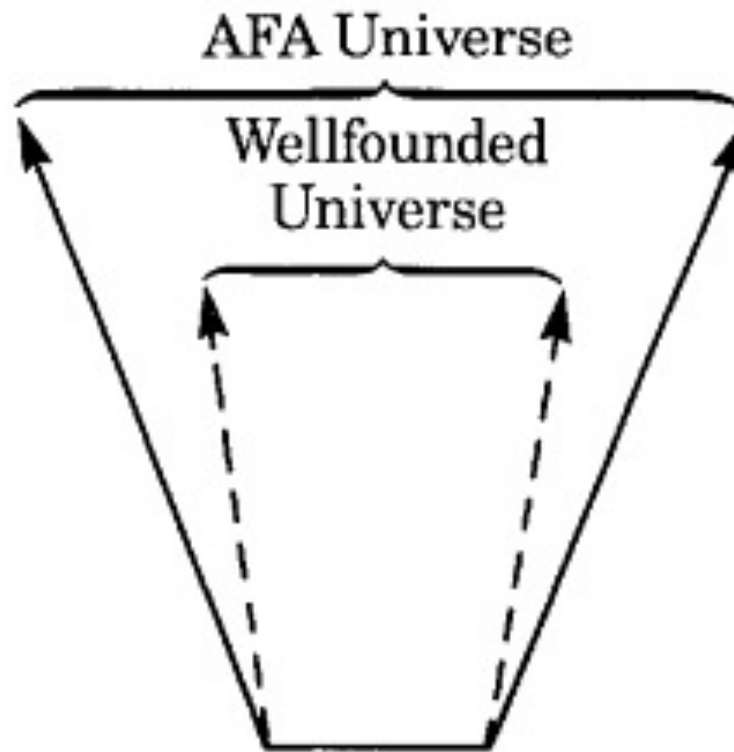
$$\dots x \in x \in x \in x \in x \in x$$

- + Can we *expand the universe* to solve the problem? Recall how a solution to the equation  $x^2 + 1 = 0$  was obtained by expanding the real number line to the complex field.

# Aczel's Anti-Foundation Axiom (AFA)

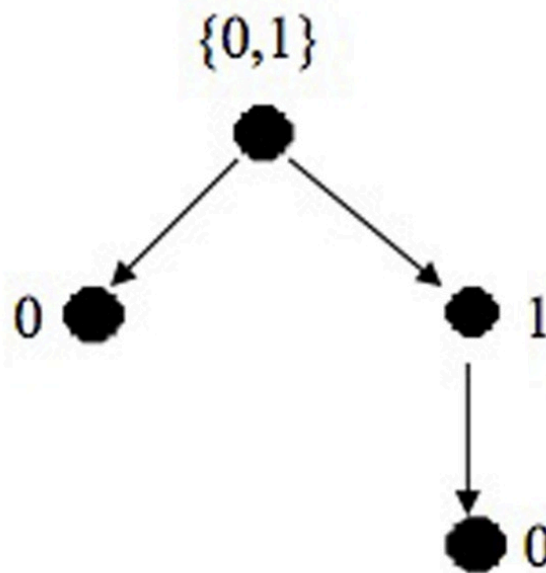
- + Can find an expansion  $V_{AC}$  of  $V$  in which there are solutions (i.e. sets) satisfying all kinds of self-referential equations, like:
  - $x = \{x\}$
  - $x = (x, 0)$
  - Simultaneous equations:  
$$x = \{0, x, y\}$$
$$y = \{\{x\}, y\}$$
- + Sets that provide solutions to such equations are called *ideal elements*. In the expansion  $V_{AC}$ , elements of  $V$  are just like before (and Axiom of Foundation is still true for them) but now there is a vast number of other self-referential entities

# The Expanded AFA Universe $V_{AC}$

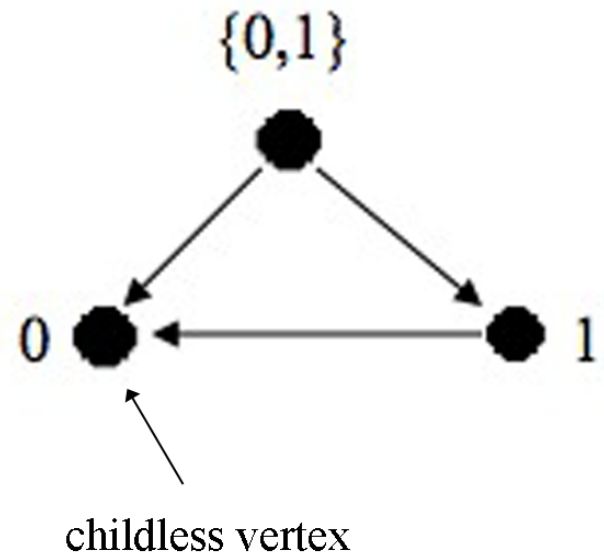
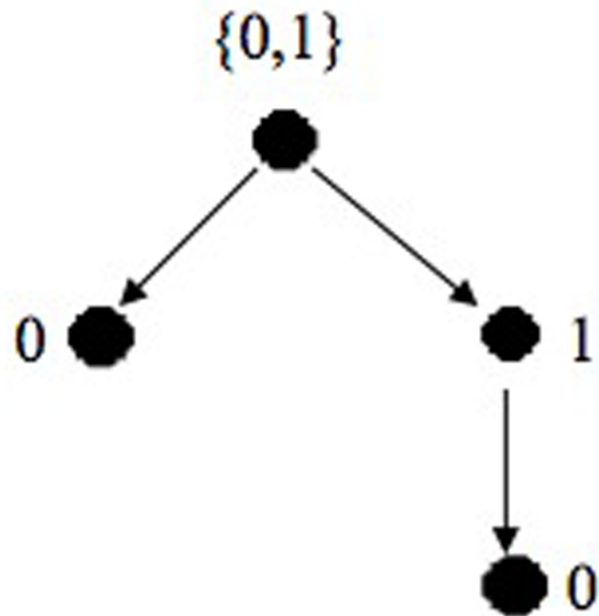


# Sets As Graphs

- + Easier to understand ideal elements if we picture sets as graphs. We will draw a graph for the set  $\{0, 1\}$ . Recall that since numbers are sets,  $0 \in 1$  (recall:  $1 = \{0\}$  and  $0 = \emptyset$ )



# The Same Set Pictured in Two Ways





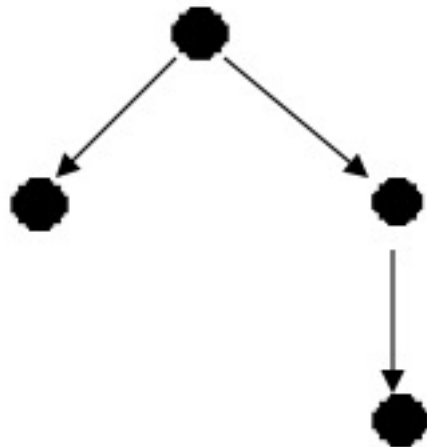
# Facts About the Well-Founded Sets

- + You can always find a way to draw a graph for an “ordinary” (i.e. well-founded) set that has *exactly one childless vertex*.
- + Although the same set can be pictured by many graphs, there is an important theorem:

**Theorem.** Every well-founded graph  $G$  is the picture of *one and only one set  $A$*  (when  $A$  is placed at the root of  $G$ )

# Example

- + There is only one set that can be placed at the root of this graph so that the rules for representing sets as graphs are satisfied -- namely,  $\{0,1\}$



# AFA As an Axiom About Graphs

AFA is equivalent to the following statement

+ **AFA Equivalence.** *Every graph (with a designated root) is the picture of exactly one set.*

This means: Not just well-founded graphs, but *all* graphs, are pictures of some kind of set, and the picture uniquely determines the set

# Example

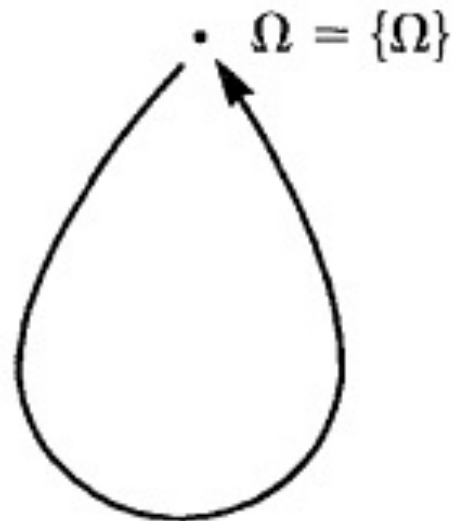
What about this graph?



By AFA, there is one and only one set that can be placed at the root. Notice that any set that is placed there is a member of itself, and is the only member of itself so...it must satisfy  $x = \{x\}$

# Introducing $\Omega$

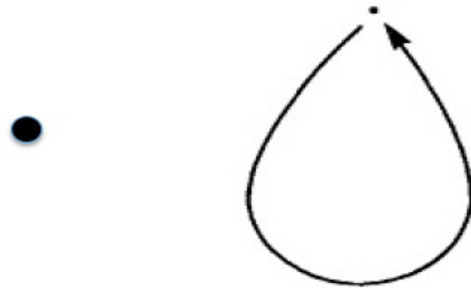
The unique set for which the loop graph is a picture is called  $\Omega$ .



Since  $\Omega$  is the unique set pictured by the graph, it is also the *only solution* to the equation  $x = \{x\}$

# Things to Notice

- +  $\Omega$  gives rise to  $\{\Omega\}$ , its own objectification, by virtue of its *loop structure*. The empty set  $\emptyset$  gives way to  $\{\emptyset\}$  because of *external operators* --  $\emptyset$  has no internal dynamics.



- + But now,  $\Omega = \{\Omega\}$  – unity is preserved as  $\{\Omega\}$  emerges from  $\Omega$ .

# Remarkable Fact: $\Omega = \Omega^\Omega$

+ In mathematics, the notation  $X^X$  denotes the set of all transformations from  $X$  to  $X$ .

+ Theorem.  $\Omega = \Omega^\Omega$

+ Proof.

(A)	$(\Omega, \Omega)$	$=$	$\{\{\Omega\}, \{\Omega, \Omega\}\}$	(B)	$\Omega^\Omega$	$=$	$\{f \mid f : \Omega \rightarrow \Omega\}$
		$=$	$\{\{\Omega\}, \{\Omega\}\}$			$=$	$\{f \mid f : \{\Omega\} \rightarrow \{\Omega\}\}$
		$=$	$\{\{\Omega\}\}$			$=$	$\{f_\Omega\}$
		$=$	$\{\Omega\}$			$=$	$\{\{(\Omega, \Omega)\}\}$
		$=$	$\Omega.$			$=$	$\{\{\Omega\}\}$ by (A)
						$=$	$\{\Omega\}$
						$=$	$\Omega.$

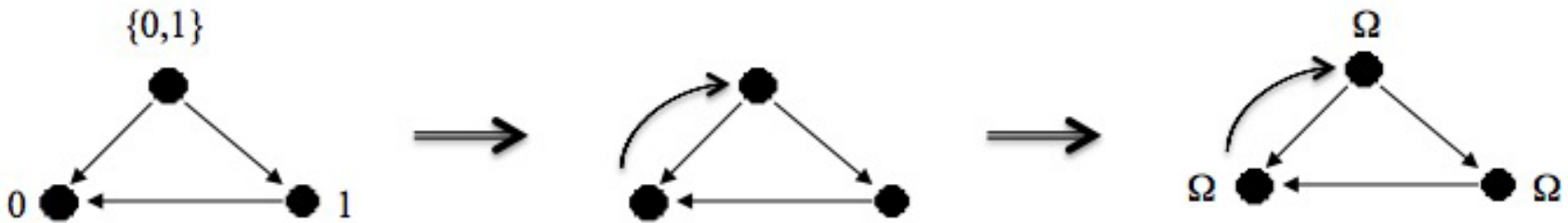
# $\Omega$ As Samhita of Rishi, Devata, Chhandas

- + **Chhandas**  $\Omega = \{\Omega\}$  Signifies that  $\Omega$  is equal to its own objectification as a point.  $\{\Omega\}$  is the “known” (**chhandas**), and  $\Omega = \{\Omega\}$  means that the transformation from  $\Omega$  to  $\{\Omega\}$  did not overthrow its unified state
- + **Rishi**  $\Omega = \Omega^\Omega$  Signifies that all transformations of  $\Omega$  occur within itself – it is the witness to all its self-referral transformations. As such it is **rishi**.
- + **Devata**  $\Omega$  is the unique transformation from  $\Omega$  to  $\Omega$ . Any transformation that occurs within  $\Omega$  is  $\Omega$  itself. Therefore,  $\Omega$  is **devata**.



# Restoring Unity to the World of Sets

- + Start with any well-founded set, pictured with a graph having only one childless vertex.
- + Add one new edge from the childless vertex to the root – this *reconnects the graph to itself*.



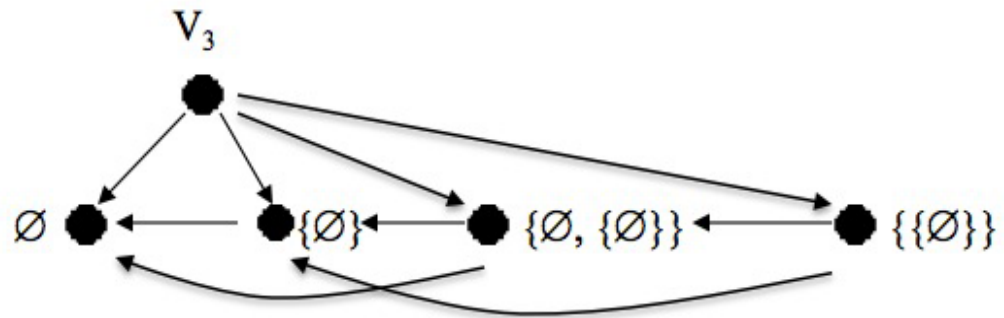
- + Adding the **Reconnecting Edge** reveals the hidden unity – same dynamics, but now completely in terms of the "Self"

# Every Well-Founded Set Has a Reconnecting Edge

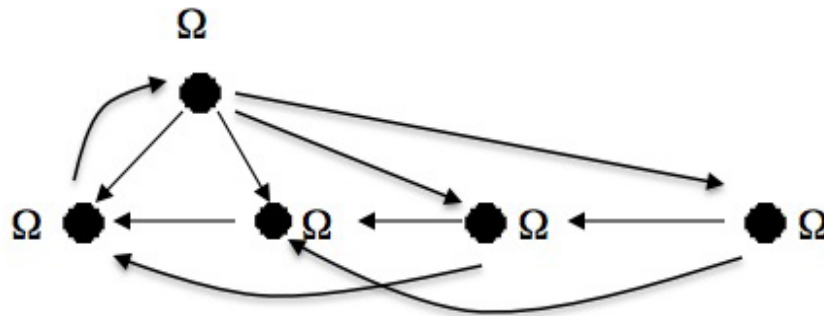
- + Adding the Reconnecting Edge *always produces  $\Omega$  and all nodes in the modified graph are necessarily also  $\Omega$* . Follows from the  $\Omega$ -Theorem
- +  **$\Omega$ -Theorem.** A rooted graph is a picture of  $\Omega$  if and only if every node in the graph has a child.

# Add the Reconnecting Edge to Each Stage of the Universe $V$

$V_3$  before adding  
the Reconnecting  
Edge



$V_3$  after adding  
the Reconnecting  
Edge

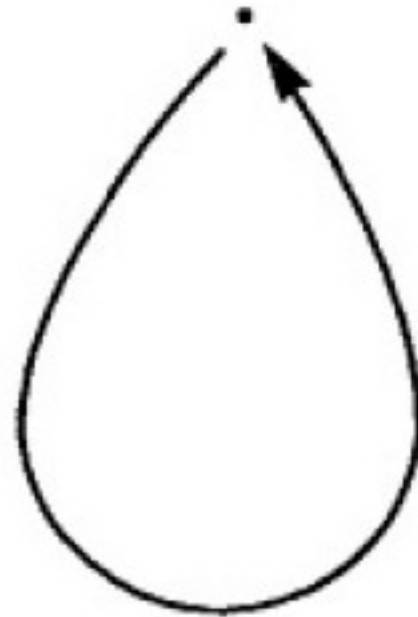


# Restoring the Empty Set to Its Full Dignity

Adding the Reconnecting Edge to the empty set transforms this



Into this



# Application to the Universe $V$

- + Can represent  $V$  as a huge rooted graph, with  $V$  itself as the label of the root.
- + Can assume the graph has just one childless vertex.
- + Adding a single Reconnecting Edge reveals all sets in  $V$ , and  $V$  itself, to be nothing other than  $\Omega$ . Dynamics within this modified universe are the same, but now everything is  $\Omega$ .

# Restoring Unity to the Universe $V$

- + Restoring unity to the sets in  $V$ , by re-introducing the Reconnecting Edge preserves the dynamics of universe but keeps the fundamental unity of all diversity foremost in awareness.

# Evolutionary Dynamics Through Self-Referral Loops

*The evolution of consciousness into its object-referral expressions, ever maintaining the memory of its self-referral source—ever-evolving structure of consciousness maintaining the memory of its source—progresses in self-referral loops—every step of progress is in terms of a self-referral loop.*

-- Maharishi, Vedic Knowledge for Everyone (p. 64)

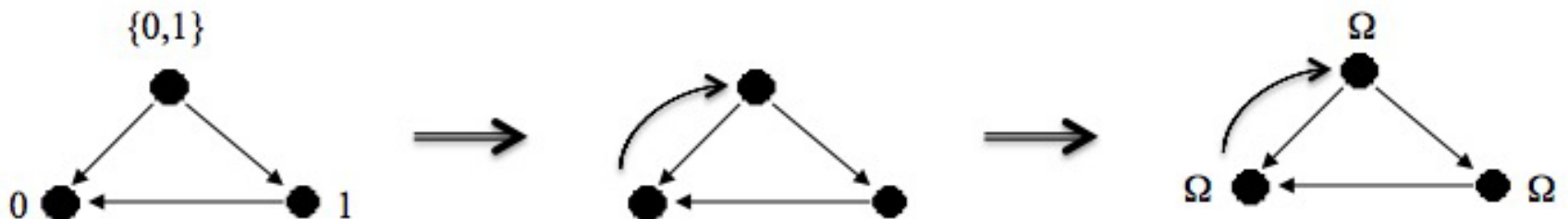
# Questions and Answers

Q & A



# Q & A. Emergence of $V$ and the Loss of Knowledge

- + All sets in  $V$  are disconnected from each other and their source. How did this happen?
- + Every set  $X$  in  $V$  can be obtained from a graph that pictures  $\Omega$  by removing a single edge from  $\Omega$  -- namely, the Reconnecting Edge.
- + Example: Do the transformations shown here in reverse:



# Emergence of Disconnected Sets

- + It follows that the emergence of the sets in our mathematical universe can be seen as the repeated act of removing the connection of sets from their source. It is the emergence of pragya-aparadh.

# Q & A: Mahavakyas

- + Mahavakyas are expressions that provide a final stroke of knowledge to bring the final stages of enlightenment to full fruition.

*Brahman becomes an all-time reality through the mahavakyas. See, through the experience everything is recognized in terms of the Self, but that experience in terms of the Self becomes significant through the teaching, because through the teaching it comes onto the level of understanding. Experience is one thing, understanding is another, and only when it comes onto the level of understanding does it become established everywhere. Then its all-pervadingness becomes a living reality . . . When the experience is ripe and the teacher says "tat tvam asi—really you are That," it's a revelation. He may have known "tat tvam asi" before, but that "tat tvam asi" did not pinpoint that experience.*

-- Maharishi, Conversations pp. 316, 318

# The Reconnecting Edge and Mahavakyas

- + Perhaps adding the reconnecting edge to the graph of V is like the Mahavakya  
*sarvam khalvidam brahma* (all this is Brahman).

# Different Graphs for $\Omega$

