MATHEMATICS

Recent Progress in the Mathematical Analysis of the Infinite

Presenter – Dr. Paul Corazza

It's really an honor to speak to this amazing group—Rajaraam, Raja Hagelin, all of you—it's really a treat. So, as Raja Hagelin mentioned, I want to talk to you about recent progress in mathematical analysis of the infinite. The main point is that Maharishi Vedic Science has been used in published research to solve a frontier-level problem in the area of the mathematical infinite, and that's what I'm going to tell you about in about ten minutes.

First I want to tell you about what we could call the "classical theory" of the infinite of mathematics. One hundred fifty years ago, it was known that if you try to list all the natural numbers, the list will go on forever, and so, in this sense, it was understood that there were *infinitely many* natural numbers. However, it was generally believed that it is *impossible* to collect together all the natural numbers into a single *set*, which could be operated on in the various ways that finite sets could be. Attempting to do this—to think of all the natural numbers as belonging to a single, infinite set—was considered taboo; there were superstitions around doing such things and religious philosophies that said it shouldn't be attempted. Set theory founder and pioneer Georg Cantor showed, finally, toward the end of the 19th century, that calculus cannot be developed properly without infinite sets; that formulating a subject so fundamental as calculus *required* the concept of the infinite set of natural numbers. So, after a long debate and an interesting piece of history that you see a summary of in the following image, infinity was finally accepted in mathematics.

- Milestone #1: (Cantor, end of 19th century)

  Infinity exists.

  1, 2, 3, . . . => {1, 2, 3, . . .}

- Milestone #2: (Cantor, end of 19th century) There is an endless hierarchy of infinite sizes

  N < P(N) < P(P(N)) < P(P(P(N))) < . . .

- Milestone #3: Formulation of the ZFC axioms of set theory and their natural model: the universe V of all sets

As you see in Milestone #1, the natural numbers 1, 2, 3, . . . go on forever, and then they are collected together into a set. Putting them all together into a single set was the crucial shift in thinking, and amounted to a declaration that *infinity exists*.

The next discovery, also due to Cantor, was that once you have one infinity, you have an endless hierarchy of ever-larger infinities. In mathematics, the letter N is used to denote the set of natural numbers, and P(N) denotes the *power set* of N, that is, the set of all subsets of the set N of natural numbers. Cantor showed that

For any set X, the power set P(X) of X is bigger than X.

So, in Milestone #2, you can see how an endless hierarchy of infinite sizes arises: We start with N, form the power set P(N), then the power set of P(N), and so forth. Each successive power set represents a yet larger size of infinity. It turns out that there are more different sizes of infinity than there are natural numbers. So the expansion of infinities really does go on and on.

We see here, then, that in the evolution of the classical theory, the first step was the discovery that infinity exists, and the second step was the discovery that the infinite has a *texture*; it has layers; there is an endless transformation within itself in the mathematical landscape.

Then, the third step in this evolution was the discovery of *wholeness* in mathematics. This discovery was made, even in the early history of set theory, because of a peculiar oversight: Cantor, the genius behind one of the deepest discoveries in the history of mathematics, somehow overlooked the fact that the concept of "set" that he was relying on was not rigorously defined. Eventually, this loose notion of "set" was shown to lead to paradoxes. The resolution of these paradoxes came about through the development of a system of axioms that declare that certain basic sets exist (like the empty set), and provide specific procedures for creating new sets from the existing sets. These axioms, known today as the Zermelo-Fraenkel axioms with the Axiom of Choice, or *ZFC*, in effect describe a universe of sets, a wholeness for all of mathematics.

The process by which the ZFC axioms were formulated is important. Since these axioms would provide a foundation for all of mathematics, it was important to formulate axioms that were *true*. It is possible to develop axioms for a system of mathematics without addressing whether the axioms are "true"; in fact, it is not always clear what "true axioms" are in certain situations. But for the foundation of all of mathematics, the starting point must be as reliable as possible. Since axioms are not *logically* derived—since they are the starting point for all derivations—it was necessary to formulate the axioms on the basis of a profound *intuition* about the true nature of sets. And, historically speaking, the primary source for this intuition came from Cantor's vision of the totality, of the *Absolute Infinite*, as he referred to it. Cantor said that the ultimate infinite, the Absolute Infinite, is itself beyond mathematical determination, but is the context in which all mathematics takes place. Using the properties of this Absolute Infinite as guidelines, together with their extensive experience in working with sets, the early founders formulated the ZFC axioms.

Experience with Cantor's vision of an Absolute Infinite eventually took shape as a natural candidate for a universe of sets, now known as the *universe V*. In these early days, some of the crucial axioms for the theory were formulated by examining whether they would hold true in this universe V. Once the axioms were fully developed, it was discovered that all the stages of the construction of V could be carried out in the formal theory. Let me tell you a bit more about how the universe V is constructed.

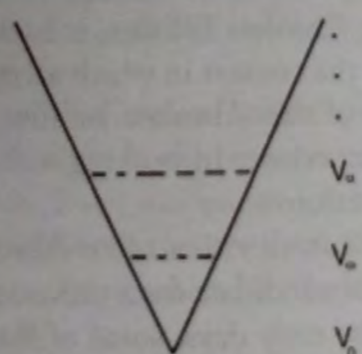
Construction of V

- V is built up in stages, beginning with the empty set, denoted V_0
- Each successive stage is the set of all subsets of the previous stage.

$$V_0 = \emptyset, \quad V_1 = P(V_0) = \{\emptyset\}, \quad V_2 = P(V_1) = \{\emptyset, \{\emptyset\}\} \ldots$$

The construction begins with the *empty set*—the set that has no elements—and each successive stage of the construction is obtained by forming the set of all subsets of the previous stage. So, V_0 is the empty set, then the next stage V_1 is the set of all subsets of V_0, which is just the set containing the empty set. And on it goes, giving rise to this ever-expanding "V"-shaped universe in which sets enter the universe ever more rapidly as we climb higher and higher.

V: The Universe of Sets



Let's look at some examples of the ZFC axioms: First, if you take two sets, you should be able to form another set, consisting of just those two sets; the *Pairing Axiom* says exactly this. Also, if you have a set, the collection of all subsets of that set should also be a set; this is the *Power Set*

Axiom. And of course, thanks to Cantor, we must postulate the existence of an infinite set; this is the *Axiom of Infinity*. It guarantees that all the natural numbers can be collected together to form a legitimate set. These are just a few examples, but they give a feeling for the axioms and how they might arise from examining the universe V.

The Axioms of ZFC and the Universe V

- ZFC axioms were formulated by studying V
- Some axioms:
  - *Pairing*: If A and B are sets, {A,B} is also a set
  - *Power Set*: If A is a set, the collection P(A) of all subsets of A is also a set
  - *Infinity*: There is an infinite set
- ZFC together with its natural model V form the foundation of modern mathematics

Therefore, the third big milestone of the classical era, in an effort to clarify the fundamental notion of a *set*, provided us with the ZFC axioms—which give us the laws that govern set formation—together with a natural model of these axioms, which we call "V." As I mentioned briefly before, the ZFC axioms are strong enough to allow us to reconstruct all the stages of V in the formal theory. This is a remarkable unification of mathematics, which already took place nearly 100 years ago. All theorems of mathematics, including the areas of algebraic geometry, differential topology, Euclid's geometry, set theory, groups, fields, rings, Lie algebras—all are directly derivable from the axioms of set theory. Moreover, every object that is studied in mathematics can be represented as a set, and every set lives in V, so what we have in this foundational ZFC theory is an incredible unification.

From the perspective of Maharishi Vedic Science, V is also a natural model for *wholeness*, for the totality. V is "bigger than the biggest" in the sense that it is too big to be a set, and it is all-inclusive since it contains all sets. Being the container of all sets but being beyond all sets, it can be seen as the unmanifest source of all sets, all mathematical creation. Because it contains all the laws that regulate the unfoldment of sets—in the form of the ZFC axioms, encoded as sets—it is a natural representative of the "home of all the laws of nature." It's self-sufficient since the encoded ZFC axioms can be used to reconstruct V within itself. It's also a field of "all possibilities," in the sense that all possible expressions of mathematical creation necessarily live in V. An application of this "all possibilities" characteristic, which we may have time to discuss during the questions and answers session, is the surprising "solution" to the problem of free will and determinism that is suggested by the structure of V—briefly speaking, one can say that the "answer" to this deep philosophical problem, which seems to arise in the study of set theory foundations, is that the points of view represented by free will and determinism are both true, and these opposite values naturally coexist within V. Anyway, more on this perspective later.

The classical era of the study of the infinite in mathematics, as we have seen, has gone down a path that is characteristic of the pursuit of the infinite in any domain, particularly in individual life. First there is the discovery of the infinite; then there is the discovery that this infinite has a character, a texture, and levels. And finally, there is the awakening to wholeness, in which the infinite and the finite are recognized as expressions of a greater totality. This is everybody's path in coming to terms with the infinite, and it's the path mathematics has taken. Initially, the infinite appeared to be just a fairy tale; it's a fairy tale for people who have never had the taste of it, and it was a fairy tale for mathematics for two thousand years. No one believed it existed. Then 150 years ago, because of the practical need to establish calculus on a rigorous foundation, the "actual" infinite was finally recognized as a reality, and the discoveries of the levels of infinity and ultimately the wholeness of mathematics in the universe V unfolded rapidly thereafter.

The *modern* era of mathematics began with the discovery of *large cardinals*. When we speak of "cardinal numbers," we are speaking of *sizes*. A set $\{a, b\}$ has size or cardinality 2; the number 2 is a cardinal number because it tells us the size of a set. We can speak of the sizes of infinite sets as well, like the set N of natural numbers, and those sizes are called *infinite cardinals*. What started happening in the middle of the 20th century was that certain mathematical problems, which had become rather prominent in the areas of algebra, topology, analysis and others, ended up with solutions that were tied to sets having extraordinarily large sizes. These large sizes of sets are called *large cardinals*. Large cardinals are infinities that are so big, they can't be proven to exist on the basis of the ZFC axioms. Of course, if they can't be proven to exist, one might wonder whether they in fact *do not* exist, and historically, many mathematicians have attempted to prove that they do not. In forty years of trying, however, by efforts from many top set theorists, no large cardinal has ever been proven not to exist. So, it's a mystery. Where are these things coming from—these big, big infinities with staggering properties that are vital parts of the research in the frontiers of mathematics?

Let me begin the discussion of large cardinals by giving you a few examples. The smallest large cardinal is called *inaccessible*. On the slide, I have presented some of the more commonly used large cardinals, listed in order of increasing strength. As you can see, an example of a "middle-range" large cardinal is a measurable cardinal; we will talk more about this kind in a few moments. Then at the upper end, you find huge, superhuge, and super-n-huge for every natural number n.

Some Common Large Cardinals

- Inaccessible
- Mahlo
- Weakly Compact
- Ramsey
- Measurable
- Strongly Compact
- Supercompact
- Huge
- Superhuge
- Super-n-huge for every n

Large cardinals seem to have a *unifying* role in the structure of the universe; bigger large cardinals introduce unification on a larger scale. ZFC itself, even without large cardinals, already brings together all the mathematics that people usually do. Nevertheless, there seem to be aspects of mathematics that the ZFC axioms do not represent very well, or in some cases, do not represent at all. Some of you may have heard of intuitionistic mathematics, which relies on a form of logic (intuitionistic logic) that contradicts the usual logic at the basis of ZFC. The usual attitude about this is that standard mathematics and intuitionistic mathematics are simply incompatible. A foundational area known as topos theory has provided a single foundational system that provides models for both standard ZFC and for intuitionistic mathematics. It is difficult, however, to represent toposes within ZFC, but if we assume the existence of unboundedly many inaccessible cardinals, topos theory can be fully expressed in ZFC. We see, in this example, that the recognition of the presence of a certain kind of large cardinal results in a unification of theories and logics that were before considered irreconcilable.

Another example of the unifying influence of large cardinals is concerned with one of the ZFC axioms called the Axiom of Choice. The Axiom of Choice says that you can form a set by randomly and simultaneously selecting elements from any infinite collection of nonempty sets. Modern mathematics relies heavily on this axiom, but some feel it gives "too much freedom" to construct mathematical objects. Some researchers began studying what the universe would look like if we replaced the Axiom of Choice with another axiom that allows a restricted degree of choice, but also imposes a high degree of determinacy; the axiom is called the Axiom of Determinacy, and it is completely at odds with the Axiom of Choice. The world of "choice" and the world of "determinacy" cannot coexist, or so it has seemed for half a century. However, it was discovered recently that, assuming the universe contains some rather large cardinals (precisely, infinitely many Woodin cardinals), it can be shown that these two worlds "peacefully coexist"—each world provides a valid viewpoint and perspective on the field of mathematics.

So large cardinals are quite interesting, but the problem is, where do they come from? Which ones exist?

And which ones may be just fanciful combinations of properties that someone invented? So, we need to know which ones exist and we need to know how to derive them. And the only way to derive them is to add an appropriate axiom to the existing ZFC axioms. If the new axiom is chosen correctly, then we can hope to derive the existence of large cardinals from ZFC, supplemented with the new axiom.

But where are we going to get that new axiom? It will require a deep insight into the structure of the universe. But again, where is that deep insight going to come from? Some of the smaller large cardinals have been justified using the insight provided by Cantor's deep vision. For instance, inaccessible and Mahlo cardinals have been legitimized to a certain extent using Cantor's insights, even though there is not, at this time, a formal axiom that everyone agrees about that could be used to derive these large cardinal notions. Nevertheless, this form of justification has been enough to give confidence to set theorists that use of these large cardinals constitutes genuine mathematics.

But what about the other large cardinals? The really big ones have no clear motivation. What we need is a deep insight about wholeness itself, which would tell us that large cardinals are really an *expected reality* in the universe V. Thinking about such things, it seemed to me that Maharishi Vedic Science would be a natural source of intuition about wholeness, which could suggest what ought to be true about existence of large cardinals in V.

If we take this approach seriously, it is natural to expect large cardinals to emerge from *transformational dynamics of wholeness*. In mathematical language, wholeness is V; transformation is represented by functions. So, we will look for a function that represents the transformational dynamics of wholeness, represented in mathematical language.

Next we need to consider what *kind* of function we will want to represent these self-interacting dynamics. First of all, we will want a function that *preserves truth*—preserves internal structure—because the transformational dynamics of wholeness in Maharishi Vedic Science preserve the fact that "wholeness is wholeness"; these dynamics don't undermine the integrity of wholeness. The second thing we want is that those dynamics should be *present at every point* in the universe. These dynamics shouldn't just be out there in some murky, undefined realm. These turn out to be two of the central features of the transformational dynamics of wholeness, as described in Maharishi Vedic Science: The move of wholeness within itself is at the unmanifest level, but yet, these dynamics are present at every point in creation. We expect those same things of our own function.

With this intuition in mind, it is natural to check to see if there is anything in the way that set theory is currently set up that would suggest that a function of this kind exists. And, rather remarkably, it turns out that our friend, the Axiom of Infinity, has, hidden within it, just such a function. Mostly, mathematicians view the Axiom of Infinity as having only a "local" character—the axiom declares that some infinite set exists somewhere in the universe. For this reason, from the point of view of the structure of the entire universe, there is nothing too exciting about the axiom, or so it seems at first. However, it was discovered about 40 years ago that this simple assertion is equivalent to a *global* statement of a very different kind, which asserts the existence of a function j from the universe V to itself. And a closer look at j shows that it has some remarkable preservation properties.

First Insight: A Global Axiom of Infinity

The work of William Lawvere (1969) shows that the Axiom of Infinity is equivalent to the existence of a map j: V -> V, where

- j = G o F
- G, F are *functors*
- G and F have strong preservation properties

Observation: We have an indication of a structure-preserving transformation. We want to give full mathematical expression to this notion. Attempt to give j itself preservation properties.

The slide shows j is a composition of two *functors* F and G. Now, a functor is simply a special kind of function that acts on all sets in the universe or in some other such vast structure, with special behavior when it acts on sets that are themselves other functions. For this talk, we won't go wrong if we just think of functors as being functions of a certain kind.

What Lawvere showed was that to assert that there is a set in the universe that is actually infinite is equivalent to asserting that there is this j: V → V, composed of two functors that exhibit strong preservation properties. The discovery of this function j tells us that we are on the right track; we are seeing signs derivable directly from the ZFC axioms that, deep in the structure of the universe, there are truth-preserving transformational dynamics at work that are intimately tied to the core activity of set formation.

Now, if you're both a Vedic scientist and a mathematician, and you are presented with these results, it would be natural for you to think, "Lawvere's result looks like a kind of broken symmetry. It looks like we almost have it, but the picture is out of balance. We would expect to locate those preservation properties in j, not just in F and G. It would appear that ZFC simply couldn't provide enough *horsepower* to give us a j that has all the properties we find in the transformational dynamics described in Maharishi Vedic Science." And so we are led to suspect that the statement that we would like to declare to be true—which *does* assert that j has all the preservation properties we could want—will require a stronger axiomatic system than ZFC.

Our plan, then, is to supplement ZFC with some assertion about the existence of a j: V → V that has more of the properties suggested to us by Vedic science. The fact that an assertion of this kind, though not quite yet what is needed, is already known to be derivable from the standard axioms and provides strong evidence that strengthenings of this statement naturally accord with the design of the universe.

To take the next step, what if we insist that j *does* have the preservation properties that F and G have in Lawvere's result? What will be the consequences? Will it be possible to derive large cardinals from such an enhancement of Lawvere's theorem? Trnkova and Blass in the 1970s found the right way to combine the properties from Lawvere's result into a single j: V → V. They showed that the existence of a j: V → V that is an *exact functor different from the identity* is equivalent to the existence of a *measurable cardinal*. An exact functor is a functor that combines all the preservation properties exhibited by both F and G, as far as that is possible. It is required to be different from the identity because the identity functor takes every set to itself, and so it doesn't really do anything; since it doesn't exhibit any transformational dynamics, it lacks the potency to give rise to large cardinals. Their result shows that the existence of a nontrivial exact functor from V to V is equivalent to the existence of a measurable cardinal, which, as we have seen, is a "middle-range" large cardinal. Now, if we replace (or enhance) the usual Axiom of Infinity by the assertion that there's an exact functor from V to V different from the identity, we will have in hand a new and improved version of the foundational axioms of mathematics that is strong enough to account for all large cardinals up to a measurable.

Second Insight: Making j *Exact*

Trnkova-Blass (1974) showed that existence of a nontrivial *exact* j: V->V is equivalent to existence of a *measurable cardinal.*

(Exactness is a strong preservation property)

Question: What if we require j to preserve the entire structure of V (rather than just *some* of that structure)?

By enhancing Lawvere's early results using the principles underlying the dynamics of wholeness, according to Maharishi Vedic Science, we have already accounted for *half* of the list of all major large cardinals. How can we go even further? While it is true that an exact functor does exhibit nice preservation properties, it certainly does not have the stronger characteristic of preserving *all internal relationships*. A transformation from the wholeness of mathematics to itself that does this could really be said to *preserve truth*, and would be a much fuller and more faithful expression of the dynamics of wholeness described by Maharishi. Functions of this kind are indeed familiar to mathematicians; such a function is called an *elementary embedding*. An elementary embedding from V to V would preserve every possible relationship between sets that can be shown to hold true in V—at least any that can be formulated using the logic of set theory. Therefore, if we "upgrade" the exactness property of j, in the Trnkova-Blass result, to an elementary embedding, we have a reasonable hope that we will be able to account for even more large cardinals.

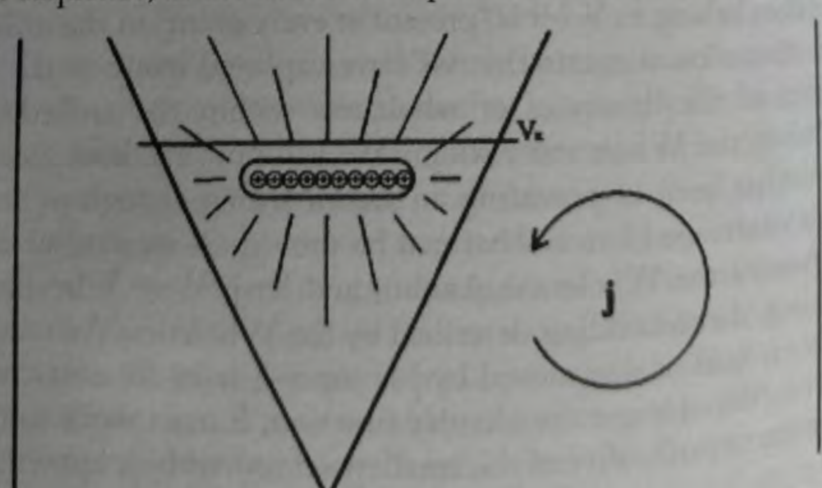
Before examining the mathematical consequences, let's introduce one more requirement on j, one that we introduced at the beginning: We will also require that j be in some sense "present at every point." This requirement is realized in a mathematical way by requiring the *restriction of j to any set to be a set as well*; in mathematical notation, we require that for any set X, the restriction j | X is also a set; this implies that the "influence of elementarity" is felt at each point in the universe.

The assertion that a j: V → V with all these properties—that j is an elementary embedding whose restriction to any set is itself a set—is called in the literature the *Wholeness Axiom*, or *WA*. There are several important points to note about the j that is given by the Wholeness Axiom. First of all, such a j *must be undefinable*. A function from V to V would be definable if there is some formula that would completely describe its behavior. The function H that takes each set x to the singleton set {x} that contains it—H(x) = {x}—is an example of a *definable* function from V to V; we can write down a formula in the language of set theory that tells you what H will do to any set. But for the j of the Wholeness Axiom, no such formula could exist. Intuitively speaking, this means that the dynamics represented by j are *unmanifest*, not accessible using the usual axioms of set theory. On the other hand, this undefinability of j does not move j into an exclusively transcendental, inaccessible realm, because we have also required that its restriction to any set does belong to V—it is "present at every point" in the universe.

Our discussion suggests that we have captured many of the characteristics of the dynamics of wholeness within the embedding j, described in the Wholeness Axiom. We can now ask how successful the effort has been in providing an axiom strong enough to account for large cardinals. Here is what can be proved, as we can see on the slide: Assume the Wholeness Axiom, and let j: V → V be the WA embedding, the embedding described by the Wholeness Axiom. Then the first cardinal or size moved by j is super-n-huge for every natural number n. Since j is not the identity function, it must move some set, and the size or cardinality of the smallest set moved by j, known as the *critical point of j*, is a very magical point—it has all the large cardinal properties on our list.

- **The Wholeness Axiom (WA)** asserts that there is a nontrivial elementary embedding j : V -> V with the property that j|X belongs to V whenever X belongs to V.
- **Theorem.** Assume WA and let j : V -> V be the WA-embedding. Then the first cardinal κ moved by j (so that j(κ) ≠ κ) is super-n-huge for every n (and more).
- The cardinal κ is obtained as a collapse of V to a point, to the first hint of transformation, analogous to the collapse of A to K.

The parallel with the dynamics of wholeness from Maharishi Vedic Science goes considerably further. This critical point of j, usually denoted by the Greek letter κ (pronounced "kappa"), is seen to emerge as a point of "infinite dynamism" within the "stillness" of j. The embedding j acts like the identity—it does nothing—until it encounters this point κ. In the first hint of transformation within V, this point κ emerges and is infused with the full power of the universe; in that "collapse," all the most powerful large cardinal properties are bestowed upon κ. Then, in some rather complex dynamics involving j, V, and κ, a sequential unfoldment emerges, known as a *Laver sequence*, which provides a blueprint for the entire universe. In fact, every set in V can be seen to arise in the interactions that occur between j, V, κ, and this Laver sequence, as the next slide represents in a somewhat artistic way.



This sequence of steps is very similar to the dynamics of "A" collapsing to "K," giving rise to the Veda, and unfolding as the universe. In these steps we see that the effort to locate transformational dynamics of the wholeness of mathematics—V—that parallel the dynamics described in Maharishi Vedic Science naturally leads to the formulation of a new axiom—the Wholeness Axiom—that more fully represents these dynamics in a mathematical context than was possible on the basis of ZFC alone, with its rather weak Axiom of Infinity. Enhancing the usual ZFC axioms by adding WA as a new axiom results in a fuller realization of the dynamics of wholeness within the mathematical universe, and results in a natural solution to the Problem of Large Cardinals. [Applause]

Maharaja: That's beautiful—that infinity can be grasped by the logic of a human being, and even accounted for—and the wholeness and its all possibilities. It's really an indication of how the human mind cognizes and trusts its own logic to see what is real, what is not real, what is acceptable, what is not acceptable, and comes out with these conclusions. Because mathematics is pure logic; it's really not based in a—you say exist or doesn't exist; it has nothing to do with the physical world. It just exists or doesn't exist in terms of logic, in terms of what makes sense or not, and that's purely the expression of that great ability that the mind has, the intellect has, to see what is acceptable, what is not acceptable, based on just logic. It's beautiful.

Dr. Paul Corazza: Thanks. One observation I would like to make about these points is that it seems that when intelligence is directed to the issue of the infinite, in whatever context it should arise, it follows the same path. When intelligence looks to the infinite in our individual life, at first it may reject it as nonsense, then it tastes it, then it explores it, then it discovers that it's everywhere. This has been the path in mathematics as well. Large cardinals originally were seen as just fanciful nonsense, and in fact, very high-level set theorists spent almost their entire careers trying to show they don't exist. Great results came from these efforts, but no one ever succeeded in proving that large cardinals don't exist. In the process, as more and more results were being discovered that were inextricably tied to large cardinals, people started realizing that these extreme notions of infinity were going to be part of mathematics now.

The next step in the process has appeared in the context of the Wholeness Axiom. From the expanded theory ZFC + WA, a rather striking and unexpected truth about large cardinals comes to light: Not only do they exist; not only do they exist in great variety; but in fact, assuming the Wholeness Axiom, we discover that *almost all cardinals in the universe are large cardinals!* And this is truly the reality of the infinite, emerging now as a truth of modern mathematics, that the infinite is nowhere absent but is rather the true nature of everything.

Dr. John Hagelin: It's really blissful for all of us to hear you speak. If you could have unfolded this talk in an hour, it would have been really fascinating to go through it at a pace that we could have followed [Laughter] more concretely. We were sort of leaping to keep up with you there. I wonder whether any of the other wonderful mathematicians in the department would just reflect on this a little bit and help tease out its significance for us.

Dr. Cathy Gorini: I must admit, I don't understand everything that Paul does, but many mathematicians will say that mathematics is the study of the infinite, and there's not really anything you can do in mathematics without the infinite. It's just absolutely essential, and the more you go into it, the deeper it becomes, and in some cases, counter-intuitive structures start to emerge. There's a famous result called the Banach-Tarski Paradox which says that it's possible to break a sphere into finitely many pieces in such a way that the pieces can be re-assembled to form two other spheres, each identical to the original. There is also the well-known Cantor set construction in which you systematically remove everything substantial from the closed interval [0,1], and yet, when you're done, there is just as much remaining as when you started. We talk about these aspects of the infinite even to our first year students, and they are able to grasp these things quite well because they have an understanding of consciousness. But it really is the case that mathematics is the study of infinite, and you just can't do anything in mathematics without it.

Dr. Paul Corazza: Let me take a moment here, if I could, to introduce myself a bit more; most of you may not know me, and so it may be difficult to know whether the strong claims that I have made in my talk are substantial and constitute "real mathematics." The points I have made today summarize work that has appeared in around ten published papers over the past fifteen years. I had the opportunity to summarize the core mathematical work in this research in a talk I gave at UC-Irvine in 2008; the talk evolved into a paper—again surveying the mathematical content of this research—appearing in 2010 in one of the most widely read journals in the set theory community. So I'm not just some crazy guy off the street. [Laughter] The Vedic Science part of this work has also been published. Several articles recently appeared in the mathematics and computer science volumes of the Consciousness-Based Education series recently published by the University.

Dr. John Hagelin: Just a quick question. Paul, you've had the chance to present these insights, as you said, to audiences of people who are in a good position to respect and appreciate their significance. This sounds like a significant advancement in a very important field—even though it may be a field that is reserved to a circle of mathematicians who are capable of thinking such things. And, of course, the inspiration for you came from being grounded in Maharishi Vedic Science of Consciousness, experientially and intellectually. What kind of dialogue and response have you had from your audiences?

Dr. Paul Corazza: Great question. It's been a very interesting experience. People started looking at the question of where large cardinals come from in the 1960s, really trying for maybe twenty years to come up with a suitable intuition and expanded foundation to account for them. But the efforts were unsuccessful; there was no general agreement even about which large cardinals should be considered legitimate, let alone the question of how to justify even those that are considered legitimate.

My own contribution to the problem began in 1990 when it was time for me to give a faculty presentation as a new Maharishi International University faculty member. I wanted to say something about large cardinals in the talk, and then the main idea about a Vedic Science-based approach to solving the Problem of Large Cardinals occurred to me. After that talk to the MIU faculty, I found a number of other opportunities to present the results.

One memorable occasion was a math conference held at the University of Florida in 1992. This was the only time that I ever dared to present the motivating insights from Vedic Science at a hard-core mathematics conference. One rarely encounters non-mathematical topics of any kind at this sort of meeting, and I was surprised that the organizers even allowed the talk to be given. The title of the talk was something like "Justifying Large Cardinals with the Wisdom of the Ancients." Two of the big names in set theory attended the talk, along with about fifty regular attendees, including graduate students and other professors. There was a lively feeling in the room as I was giving the talk, and when I finished, there was an enthusiastic standing ovation. It was as though the talk had touched some nerve; as though, in some unexpected way, the participants were moved by this connection that was being brought to light, between ancient threads of wisdom and the modern treatment of mathematical infinity.

Another important development took place in the year 2000. In that year, I saw that a lot more could be said about this Wholeness Axiom. In those days, there was a commonly held belief among set theorists that it really was not possible to have an elementary embedding from the universe V to itself. The idea that such an embedding would represent, in a sense, the "ultimate" large cardinal axiom had been recognized as early as the 1960s. But a major researcher in the field, Ken Kunen, proved—or so it seemed—that the existence of such an embedding was inconsistent with ZFC. Even as a graduate student, I felt certain there had to be more to the story, that Kunen's result could not possibly be the last word on it. It seemed to my naive way of thinking at the time that Kunen's result was in direct contradiction to Vedic wisdom! As my experience in this area of mathematics grew, I was able to examine Kunen's work more carefully, and eventually I found the "flaw" that I was certain had to be there. Kunen did indeed prove something very important, that under certain conditions, an embedding $j: V \to V$ of this kind really is inconsistent. But my work showed that one cannot conclude from Kunen's proof that such an embedding is actually inconsistent with ZFC (and, interestingly, Kunen himself never did draw this conclusion; it was just a popular interpretation of his work). My approach in devising the Wholeness Axiom was to investigate what happens if we do *not* make those extra assumptions that would lead to inconsistency.

So, in the year 2000, I gave a talk at one of the big annual conferences—an event that typically attracted the big names. The title of my talk at this conference was "Rumors that Elementary Embeddings from V to V Don't Exist Have Been Greatly Exaggerated." For set theorists at the time, a title like this was flashy and kind of irresistible. It was a bold claim because everyone knew about Kunen's result, and almost everyone interpreted this result in the same way. Many of the big names came to the talk, from both philosophy and mathematics departments. At the conclusion of the talk, one of the top set theorists asked me more about the implications of the Wholeness Axiom on the structure of the universe. He asked questions that were reasonable, and although I believe I had answers to those questions, he was not really convinced.

Over the years, I have had the chance to discuss the Wholeness Axiom with many of the leading researchers both at conferences and one-on-one; mostly the feeling about it has been that it's a good approach, though one or two of the really top people don't like my approach at all and prefer to think about the issue of large cardinals in a different way altogether. By the time I gave my University of California-Irvine talk in 2008, many of the leading researchers had become aware of my work and a fair number liked certain technical aspects of the work—not necessarily the Wholeness Axiom itself, but a number of other results that were connected to it. During questions and answers at the end of the talk, questions came from just two people. Both were among the very best in the field, and, to my surprise, the tone of their questions was very respectful and showed genuine interest in where the research would go from here.

The situation today is that this work on the Wholeness Axiom, for those who are aware of it, is viewed as one of several approaches to justify large cardinals. But by now, interest in finding an answer to the Problem of Large Cardinals has waned considerably. I think the fact that nothing conclusive came of the intensive study of this question that took place roughly between 1965 and 1989 led to a certain apathy about the whole issue. By the 1990s, researchers were no longer looking for a foundation for large cardinals. The issues are still there but people just stopped caring, or at least that's my assessment of it. Honestly, I don't know how you could *not* care. [Laughter] ZFC was found to be an axiomatic system that provides a basis for all of mathematics, a great unification. Then these large cardinals come along that cannot be derived from this unifying theory, so of course you would want to expand the foundation to provide an account for these other mathematical entities. But, I believe, people just got tired of working on it. So, I think, the solution I have come up with came a bit too late in the game, at a time when the Problem of Large Cardinals was no longer a problem people were trying to solve. So, my work is respected, but it isn't going to become a new paradigm, a new foundation, at least not right now. [Applause]

Dr. David Streid: One comment I'd just like to make is that Paul is clearly absolutely brilliant on these things. It also reflects back, I think, on the conversations we were having earlier in the day on the nature of the mind and its ability to know the universe. Ultimately, the way we get these axioms in mathematics is through self-referral consciousness, by going deep within. The people explore those more refined levels of consciousness. This is essentially what Cantor did; he had that experience and was able to have that self-referral process which led him to discover and analyze the infinite in a different way than other people had. This is essentially what Paul has done in his research as well. Mathematics itself, through Gödel's work, has really shown us that this is in fact the only way that we can do it; that we can't do it solely on the level of logic. He showed that if we try to completely axiomatize the truths of the universe on the level of deductive logic alone, there will always be something left out; we can't have an absolutely complete theory on that level. It's only by going to a deeper level, a self-referral level of consciousness, that we can have that complete knowledge. [Applause]

Dr. Greg Guthrie: I want to comment that Paul also has a grounded side working in the computer science department, teaching the applied side of this, so he's at the top level of abstraction and the most applied, practical level as well. He spans the whole gamut. I was thinking about

Raja Hagelin's comments about the more and more abstract levels and these incomprehensible numbers like 10^{100}—I tried to write this down but I ran out of space as I was writing all the zeros—these are such huge numbers, and yet they're *finite* numbers. Yet, when you look at these large cardinals and these large structures of infinity, and structures and the textures—the description you gave, Paul—I wonder whether those same sorts of structures, and perhaps the concepts of that mapping of infinity into itself, and the functors that map and stay within the same domain and all that—all of those same structures of natural law, conceptual though they may seem in mathematics—would also show up somehow at that most refined level of understanding in quantum physics. Is there a terminology or mathematical model shared between those levels?

Dr. Paul Corazza: Well, I wish I knew what Raja Hagelin knows—I think most of us do—because then I might be in a position to make those connections in a more substantial way. I've taken John's physics course so I have learned some of the very basic elements of modern-day string theory, and, from my naive point of view, I see certain parallels to the world of large cardinals. One parallel that is interesting to me is the way that, as one goes "deeper," one discovers greater unification of apparently distinct and divergent aspects of the universe. In the case of quantum field theories, greater unification is found as we go to smaller time and distance scales; at the Planck scale, finally, everything is seen as the self-interacting dynamics of a single field. In the world of set theory, unification is seen to occur as we rise to ever larger large cardinals. As we accept bigger and bigger large cardinals into the universe, more and more we find that competing theories can, in a sense, peacefully coexist, whereas without large cardinals, they are seen to be irreconcilable. Examples include the topos-theoretic models of intuitionistic theories and the canonical model of the Axiom of Determinacy, neither of which is really consistent with set theory in the absence of large cardinals.

Whether one can actually take an analogy between modern string theory and set theoretic foundations and derive some technically correct conclusions—conclusions that might conceivably lead to interesting and even a bit bizarre consequences, like the possibility that the presence of large cardinals in the mathematical universe has certain implications for our physical universe—whether something like this is really possible I still don't know.

I was contacted several years ago by a physicist who had studied my work on the Wholeness Axiom. She said she was attempting to formulate quantum mechanics from the ground up, starting with the mathematical world of ZFC + WA. This seemed like an exciting project to me, but it was outside my range of expertise. I think she got a few physicists interested in her work, but I don't think she got as far as she had hoped.

Dr. John Hagelin: One very quick comment. Maharishi was describing to us in those last days—as was reported this morning—that he was "floating in the unfathomable, unfathomable ocean of pure Being." The way he was speaking of it was in a state of wonder. Here is someone who is living Brahman Consciousness already[11]—moved to a state of wonder. We saw a glimpse from the standpoint of the deepest, deepest, most abstract and unifying physical ideas earlier this morning. And just a small little picture of it here and there, how it appeared as the infinity of universes, is truly a huge reality—and so sublimely abstract and comprehensive in nature that it's uniting, falling into place like pieces of puzzles. The very unimaginably diverse areas of physics and math are inextricably and intimately unified.

As I said to Cathy Gorini, who spoke briefly, to move this field forward is really going to require the combined intelligence of all the mathematicians and all the physicists—but most importantly, those who are steeped in the experience of the absolute unbounded, because Maharishi Vedic Science is illuminating the cutting edge of modern science. Any restriction in the size of our own comprehension is just not going to grasp it. Certainly, following Paul's talk would become quickly challenging.

So, that sense of the unfathomable—unfathomably huge, comprehensive, and inclusive—shines through your talk very much. And really, the limiting factor in all of this is the ability to fathom levels of expanded comprehension that are so hugely unlimited by anything. Transcending isn't something you do only once after your second night

[11] Brahman Consciousness is a higher state of human conscious in which the knower experiences the totality of existence through self-referral consciousness.

checking and you're done. Rather, the fathoming and expansion of deeper and deeper levels of unbounded awareness is a process of culturing that experience and stabilizing it.

So I think people progress to the "height of their incompetence." And what that means, when you're studying physics, is that you study this level of math and this level of math, and suddenly you say, "Oh gosh, this is hopelessly abstract and irrelevant. I'm going to major in home economics, because in home economics, I can deal with 1, 2, 3, 4 and don't even need calculus."

So it's just a question of the degree to which our awareness is stuck at what we call our conscious thinking level, concepts bigger than which—more abstract, and comprehensive than which—we don't fathom due to the blinders we wear. Letting those blinders relax and open up and expand by steeping ourselves in unbounded awareness is the mathematicians', physicists', and probably all academics' number one tool that Maharishi has given to us. That's why, I think, this group has made and will continue to make the cutting-edge discoveries and advances in the field. [Applause]

Maharaja: Beautiful. Thank you.

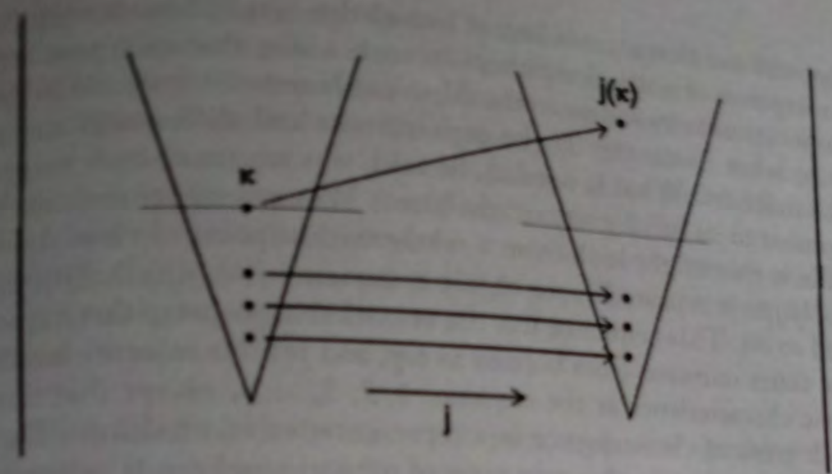
Dr. Anne Dow: I would like to also bring the idea of infinity a little closer to home for some people in the audience. The point I want to make is that infinity is there whenever we let a variable x stand for all possibilities. Then we put it in an equation and solve for it and find out definite values for it. In this way, we collapse all possibilities to a point value. This is what happens when we solve an equation, and usually this is done in the context of some application of mathematics. So this expansive view of all possibilities, of infinity, is already there at a very elementary level of mathematics.

Another way all possibilities shows up at an elementary level is in geometric constructions. We now have more than one type of geometry. We have all the different kinds of geometries that allow us to think of space as a surface, or a saddle, or something like that, where there are different natural laws, where, for example, the sum of all the angles in a triangle add up to something different from 180 degrees. So, there are all possibilities in the area of what kind of structures we can have that we can apply in the world, and there are all sorts of possibilities just from ordinary equations like $3x + 7 = 27$.

We talk about these perspectives on infinity even in our elementary algebra classes. It's all there. So these points just bring a more elementary face to this discussion about infinity. And even though they arise in an elementary setting, they can be quite high levels of infinity. We have the real numbers that exceed the counting numbers in size, and they can be solutions to equations, these simple equations, so there's a larger infinity there at the very base.

I wanted to come back to a question that I haven't given you any warning for, but I have a deep feeling about the idea that Maharishi has brought out—of the Absolute Number being connected to the unified field; that it is embodying that pure intelligence, and it unfolds from that. I can understand all numbers as being in some sense absolute, but I think there was a time when Maharishi felt that there might be some one thing in mathematics that could be considered the Absolute Number. I'm not sure if we've ever uncovered that.

Dr. Paul Corazza: I was here on faculty when Maharishi was raising these points with the mathematics faculty, and I offered my own response to this question he was asking. Let me tell you this point, which seems to follow naturally from this new framework we have been discussing. This Wholeness Axiom says that there's a particular place in the universe where silence first steps into dynamism. We can see it on the slide (below). In the diagram, we see the universe V, mathematically representing wholeness, and j, representing the transformational dynamics within wholeness. As we examine the behavior of j, we see that at the beginning, it does nothing; it is just silent, not acting at all. Each point in the universe at these earlier stages is taken to itself by j: j is just the identity function. Eventually a point arises, which we call κ ("kappa"). It is one of the hugest conceivable large cardinals—for the moment, it's fine to think of it as some enormous number. When κ comes into view, j starts to act, starts to move—it sends κ to some other cardinal, some cardinal j(κ) having the same strong properties as κ, but now j(κ) is even bigger.



When κ moves, it is as if empowered with infinite dynamism. We had infinite silence, nothing happening, and then this point κ comes into view—the first hint of transformation—and it is imbued with incredible power. It is powerful not only because it has all large cardinal properties, but also because it gives rise to all sets in the universe. This fact comes about because, from the interaction between j and κ, there emerges a special sequence of sets—a *Laver sequence*—and, through the interaction between j, κ, and the Laver sequence, every set in the universe can be specified. This means that all sets in the universe arise from κ, emerging as a point of infinite dynamism from within j's completely silent phase of operation as an elementary embedding. In this sense, κ speaks for all sets.

In fact, κ can even be said to speak for the wholeness of the universe as well. Recall that the universe V is built in stages V_0, V_1, V_2 . . . Eventually we get to the κth stage, V_κ. It turns out that V_κ is an absolute reflection of V. What can be shown is that everything that is true in V is also true in V_κ, and conversely. Every mathematical assertion that holds in one holds in the other. And V_κ can itself be coded in κ. This tells us that everything about the universe, all knowledge of the universe V, of wholeness, is located in this point κ. In this way, κ is a natural representative of wholeness within the realm of sets; it is a set that speaks for wholeness.

One can say somewhat more, in the spirit of Maharishi's Absolute Number. In his discussions about the Absolute Number, Maharishi pointed out that a great loss of knowledge resulted in viewing the emergence of natural numbers in such a way that each number's connection to its source in the Absolute Number is lost, and in this way, what dominates is the separateness and differences among the numbers. What is needed, he said, was to restore each natural number to its full dignity in wholeness. We can now see indications of how this might look from a mathematical point of view. As we apply j to κ repeatedly, we obtain a sequence j(κ), j(j(κ)), j(j(j(κ))), and so on. This sequence has the remarkable property that it spans the entire universe, from bottom to top, and yet this sequence has the same characteristics as the sequence 1, 2, 3, . . . , except that now, each term of the sequence is a representative of wholeness. Each natural number, in this new view of natural numbers, is automatically appreciated in terms of wholeness. And although the terms of this sequence are distinct, wholeness dominates; one can show that they collectively exhibit the property of *indiscernibility*—their differences cannot be detected by first order formulas. [Applause]

Dr. John Hagelin: What I saw from this talk was that there is a level of intellectual activity that captures or gives expression to infinity, or a level of intellectual activity at the level of mind or intellect that is fathoming the unfathomable, the wholeness of the infinity of sets in a sense; it's mirroring it perfectly. Maharishi talked about the Bhagavad-Gita once as the highest expression of intelligence that was understandable by man. But then he exposed us to other aspects of the literature, like Karma Mimamsa, the Jaimini Sutras, and said, "Don't bother trying to understand it; the logic is sound, but it's not a human level of logic." And it certainly wasn't anything I was ever able to comprehend. But its effect, nevertheless, was a concrete effect created by swinging in those waves of logic.

So it may be that there are levels of development not normally considered to be human intellectual endeavor that are really on an absolute level of mind, or a higher level of intelligence. Krishna actually said something to Arjuna, as I remember, that this is a revelation that humans can't comprehend.

I don't know. The idea is sort of speculative, but it's interesting. The key point that strikes me was what you said: there are levels of flights of incredible, expanded comprehension that mathematicians juggle with an incredible agility. I must say, they really are able to capture the unbounded, in a sense—maybe more profoundly than M theory is beginning to, in that it seems to be able to explain everything. And we can only just vaguely grab hold of it, but it seems to be a reflection of the absolute, and of sufficient power that it is capable of unifying all knowledge.

It seems to be.

Dr. Paul Corazza: It certainly was such a pleasure speaking with everyone. Thank you so much.

Dr. John Hagelin: Tomorrow's schedule includes some amazing areas that we'll be delving into: Neuroscience, which will be launched by Dr. Fred Travis; Medicine and Physiology – Part 1, presented by Dr. Robert Schneider in his "Epigenetics and Consciousness: Modern Science Discovers How to Modify Your Biological Destiny"; and after lunch, Medicine and Physiology – Part 2: Dr. Keith Wallace's "Personalized Medicine: Epigenomics and Ayurveda." Later in the afternoon: the fascinating area of computer science with Dr. Bruce Lester and Dr. Greg Guthrie. Maharishi was so fond of Dr. Bruce Lester's original Unified Field Chart—I think it was the first one of them all—and some of the beautiful knowledge of computer science that Dr. Lester brought out with Maharishi so many years ago.

Dr. Keith Wallace, our University's founding president, will be the referee tomorrow, to be sure nobody violates the rules, as I have tried to be today. So that's what's coming up tomorrow; it's going to be an amazing day. Also, these are areas that Rajaraam is profoundly at the top of in terms of expertise—medicine, neuroscience; it's going to be an amazing dialogue. But anything you'd like to leave us with as we close?

Maharaja: It's beautiful. If you have questions regarding what has been so far discussed, maybe we can pass them on to somewhere, and . . .

[BEGINNING OF MORNING SESSION]

Questions from Previous Day's Sessions

Dr. Keith Wallace: Welcome to our second session of the Faculty Seminar with Maharaja Adhiraj Rajaraam. It's a real joy to have him here and be able to discuss all our topics. We thought we'd start this morning just for about the first five or ten minutes with questions that were asked from the previous session. We'd like to ask Maharaja to answer a few of these questions if he would.

Maharaja: *Jai Guru Dev*. I'll address that first question from Dr. Streid, the one related to the infinities. Dr. Streid summarized the presentation yesterday from Dr. Corazza, our great mathematician with infinities, and he said that, "Cantor described three kinds of infinity—potential, actual, and absolute—and I wanted to know how we can look at these three." There is a proposal that perhaps they correspond somehow to the states of consciousness or maybe the actual infinities correspond to, one way or the other, to Cosmic Consciousness. Absolute infinity would be Brahman, and the potential infinities are what we experience in terms of the sequence of continuing numbers and expansion of the universe and all the possibilities that can emerge from it.

This question gives us the opportunity to look into states of consciousness. Without going into too much detail, states of consciousness are actually states in which reality is different from one state to the other. As we say, reality is different in different states of consciousness. That means that when you are dreaming, you are dreaming. When you are sleeping, you are sleeping. When you are waking, that's the reality and how you perceive it. When you are in Transcendental Consciousness, there is nothing but pure Being. In Cosmic Consciousness, you do see these two values. So that is right: One can think that actual infinities can be like this, perceived from the knower's side. I would rather, if I understood the concept of infinities well, put them into actual realities because it seems to me that they coexist. Is that true or not? That the three types of infinities, or classes if you like to call them, potential, actual and absolute, actually coexist at the same time? Or are they just different points of view about reality?

Dr. David Streid: They're different types, and we can look at them in different ways. Some people have felt that you can't have them in certain types, but Cantor's basic discovery was "Yes, we can."

Dr. Paul Corazza: You're right, the actual infinity and absolute infinity do coexist in the mathematical point of view. Actual infinity is a deeper insight into potential infinity from the mathematical point of view. Potential infinity was just all that people could grasp and so potential infinity has been upgraded to actual infinity in mathematics. In mathematics, there's no such thing as potential infinity; it's just actual.

Maharaja: Great. If we look at it from one perspective at least—one can look at things from different perspectives—Dr. Streid wanted to know how the intellect appreciates it. Particularly in the actual infinities there are hierarchies, and so you can go in the hierarchies to the smaller and the smallest, and then you probably find even infinities in that direction, or you go into the higher and higher, and then in these hierarchies, you will have even bigger infinities. Is that the case?

Dr. Paul Corazza: It's really an interesting question. Certainly, one could philosophically think of going smaller than the smallest. The reality of set theory is that it doesn't go any smaller than the empty set, which is zero, so we don't get infinity in the other direction, except there are certain kinds of nonstandard mathematics where you do that. My insight, whether right or wrong, is that if you want to get bigger than the biggest, look at mathematics. If you want to get smaller than the smallest, look at nature, and small time and distance scales.

Maharaja: You should be careful when you speak to a mathematician because mathematics is very precise. I actually didn't mean to really go smaller than the smallest, just in the direction of the small. Thank you for really being precise. [Laughter] My feeling about it, and that's why it's really nice to start with it, because it brings into consideration—and I like to hear your opinions and everyone else's—about this way of looking at it. See—if we go back to our basic one unbounded infinity, which we can say is the Absolute, pure unified field, and we say this is the absolute unity—infinity, well, we say that's the absolute infinity. It is Absolute, meaning that it is non-changing. It is equal to itself. It is beyond time and space, and it is one absolute infinity. If we acquire this as one reality, we all can understand it intellectually because we have the experience of it, and we have the concept of it through the physics of the unified field.

Can we accept that so far? Good. Then what we said is that how the universe manifests is actually that the same Absolute has so many potential forms of expression that the potential infinity becomes all the potential ways it can become, or be, or expand, or evolve, or change in time and space, in structure, in everything. All these potentialities would become potential infinity. Can we can accept this also?

Dr. Paul Corazza: It's a beautiful point. Of course, it wouldn't correspond to potential infinity in mathematics so much. I mean, the same insight is there; it is just in the wholeness of the universe. Every possibility is there, is present. Different points of view within that universe allow anything to be realized, but they're all actual infinities.

Maharaja: They are all actual infinities?

Dr. Paul Corazza: Yes, that's all there is.

Maharaja: But the potential is the potential for them to exist, we can say.

Dr. Paul Corazza: Well, in mathematics, "potential" is represented by consistency. You can have a sub-universe where one thing happens, another sub-universe where something else happens, so you can say that those things are potentialities, possibilities. For example, there is Cantor's famous Continuum Hypothesis which says that the size of the set of real numbers is the second largest infinite size. In some universes it's true; in other universes it's false. Potentially, either possibility could be true, and both possibilities are integrated as possible realities in the universe.

Maharaja: But these potential infinities do become actual infinities themselves? Or is that a different class?

Dr. Paul Corazza: Well, these potential realities—if you step into a universe—they're realities. If you step out of that universe, they're no longer realities.

Maharaja: So there we have a point of view from where you're stepping.

Dr. Paul Corazza: Yes, exactly, different points of view. So, potential, what you're calling potential would be different points of view.

Maharaja: Because I was going to propose that the actual infinities in reality are everything we observe. Maharishi explains how in Veda we see the reality of a point and the whole, point and wholeness, infinitely small and infinitely big—or zero, of course, as a point, a kind of zero dimension and infinity. In Vedic Science, we often say, that every point contains infinity.

Now, the point is one point, which is non-dimensional, a non-dimensional structure, neither in time nor in space. It is how I think a mathematics point is defined, to some extent. In reality, when we speak from the Vedic perspective of any point, it's actually any object, any structure, any aspect of the universe that we can define, even in time and space, because if you say that this microphone is not a point, it is a big structure, but from the perspective of the Absolute, it is a point. Now the body is a point. The room is a point. The whole earth is a point—with respect to the Absolute, with respect to wholeness.

The universe, even the created universe, can almost be seen also as a point. If we consider these that are the actual realities and analyze them, what are they made of? Say they are made out of the unified field. We are saying that the point contains totality, contains infinity. Could it then be understood on an intellectual level that these different points, which have different hierarchies of dimension when perceived from the observer's point of view—because an atom is a tiny point; a planet Earth is a huge point—could these be called points because they are relative structures in the Absolute?

Now, they are hierarchically different sizes from a certain point of view, but in reality they all contain infinity. So the point, which is that atom, contains or is the entire unified field, in a sense, and the planet

Earth is another way to look at the expression of the unified field, from another sense. Here, on an intellectual basis, one can imagine that, in fact, the Absolute has wanted to manifest itself, as we said, in order to say, "I am nothingness. I am wholeness, but I am all possibilities; therefore I contain within me the option of nothingness." This is how one can imagine the Absolute, pure, unbounded consciousness floating within its feeling of being absolute and realizing it is all possibilities, and then suddenly realizing it is a point; it is nothingness.

This is like concentrating all of this huge unboundedness into a point perception. When you concentrate something into a small point or when you put pressure and condense it, even as you condense air in a tire, it heats up. You know the process when compressing things, and that's how probably the heat comes, and the explosion comes, and the Big Bang. This Absolute, being so unbounded, compresses itself into a point, and the heat is so huge that it explodes into the Big Bang and into the whole creation.

If we want to add a purpose to it—that would be that it would be expressing itself into so many infinities to show to itself that it is not a point but even so many more infinities that you imagine or potentially create. But still all of these points that are created are points, from a certain perspective, and they are at the same time infinities. So the Absolute is as if the Creator—if you like, just to make it more personal—expresses himself or herself through his or her creation, or both together, and they create, or it creates. It creates itself in so many infinities that have hierarchies of consciousness, hierarchies of volume, hierarchies of distances, hierarchies in space. There are all of these potentialities of actual infinities, but they are still hierarchically conceivable as being different levels of infinity. Can this description be satisfying?

Dr. David Streid: It is, and there is one other perspective that I think is a very nice explanation that I've tried to use in the past. It is Chapter 6, Verse 3 of *Maharishi Mahesh Yogi on the Bhagavad-Gita, Chapters 1–6*. Maharishi talks about the transformation from Cosmic Consciousness to God Consciousness as being the transformation of silence, and he says that, "The silence which is experienced in Cosmic Consciousness, and which separates the Self from activity is on an infinitely smaller scale [than that of God Consciousness] for it is on the level of individual existence. The one forms the basis of the activity of the whole of creation, the other the basis of individual activity" (289–290). So in a sense that kind of is in the same direction; it's the size of the point.

Maharaja: Yes. Beautiful.

Dr. Paul Corazza: Everything that you said was just great. I just thought it was great. I'd like to see if I could make the same point in the context that I understand. I know that the discussion about large cardinals went by too quickly and the points about it may still seem a bit out of reach, but let me just summarize the main point by saying that once you represent in mathematics the idea of transformational dynamics of wholeness—that it is actually the nature of wholeness to have those dynamics—using this idea of an elementary embedding j from the universe (wholeness) to itself, then you can look to see if what happens in the mathematics of it is like what Maharishi Vedic Science predicts.

Sure enough, when such a j is invoked, all the properties of wholeness—the universe V—get concentrated into a single point in the universe, which we have been calling κ. That point represents wholeness in the sense that everything that is true about the universe is true in that point. Wholeness is there. There are various ways to demonstrate that. What happens with all that concentrated power in that single point is that something emerges sequentially. It's called a Laver sequence, and that Laver sequence encodes the entire universe; all sets in the universe can be seen to emerge from it. It's like the Veda. It's a sequential unfoldment that comes from a concentrated power of that single point and everything in the universe emerges from that single sequence, and all the hierarchy of infinities results from that, from those dynamics.

Everything in the universe essentially is infinite and almost everything has a large cardinal power. It's that strong. If you don't know about wholeness and its dynamics, everything appears ordinary, finite, limited. But once you understand that wholeness is really a reality, and that it has these dynamics, then everything is a large cardinal practically, everything is filled with wholeness. [Applause]

Awakening From
Ignorance to TC to Unity to Brahman

- Step 1: (19th century mathematics) "Nothing infinite exists"
- Step 2: A taste of the infinite (N exists)
- Step 3: The infinite has different sizes and expands within itself–it has a *nature*
- Step 4: "Large cardinals are an ...usion"
- Step 5: Some large cardinals seem to be real, legitimate
- Step 6: (WA) Large cardinals arise from wholeness interacting with itself (Unity)
- Step 7: (WA) *Almost all cardinals are large cardinals* (Brahman)