Recent Progress in the Mathematical Analysis of the Infinite

The Classical Theory

- Milestone #1: (Cantor, end of 19th century)

  Infinity exists.

  1, 2, 3, . . . => {1, 2, 3, . . . }

- Milestone #2: (Cantor, end of 19th century) There is an endless hierarchy of infinite sizes

  N < P(**N**) < P(P(**N**)) < P(P(P(**N**))) < . . .

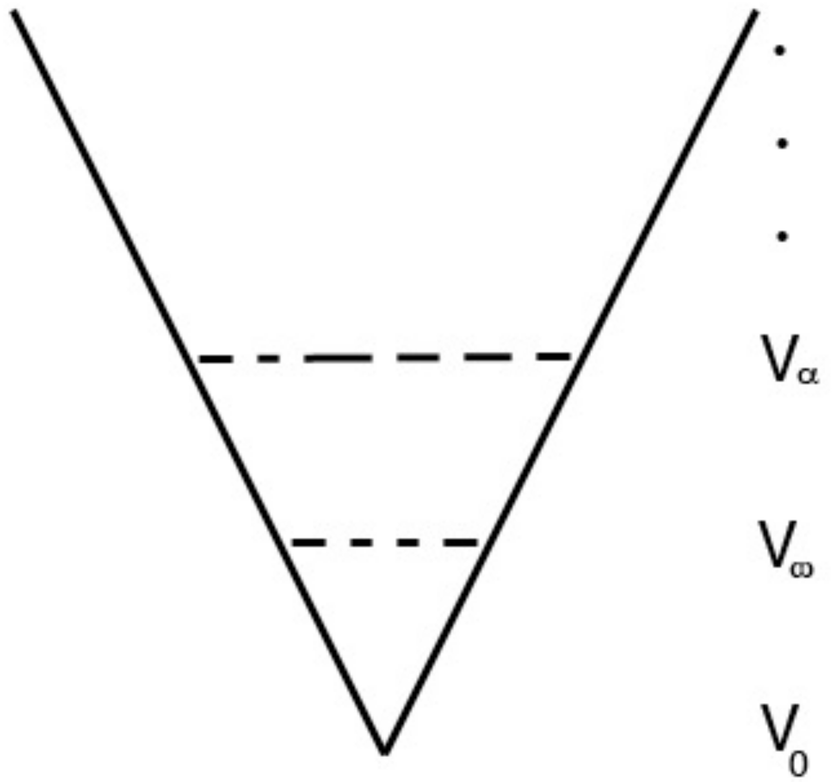
- Milestone #3: Formulation of the ZFC axioms of set theory and their natural model: the universe V of all sets

Construction of V

- V is built up in stages, beginning with the empty set, denoted V_0
- Each successive stage is the set of all subsets of the previous stage.

$$V_0 = \varnothing, \quad V_1 = P(V_0) = \{\varnothing\}, \quad V_2 = P(V_1) = \{\varnothing, \{\varnothing\}\} \ldots$$

V: The Universe of Sets



The Axioms of ZFC and the Universe V

- ZFC axioms were formulated by studying V
- Some axioms:
  - *Pairing*: If A and B are sets, {A,B} is also a set
  - *Power Set*: If A is a set, the collection P(A) of all subsets of A is also a set
  - *Infinity*: There is an infinite set
- ZFC together with its natural model V form the foundation of modern mathematics

The Modern Era

- At the beginning of the 20th century, extremely big cardinal numbers began to emerge in research. These came to be known as **large cardinals**.
- Kurt Gödel (1930) proved that

It is impossible to prove from ZFC
that large cardinals exist

- No one has ever proved from ZFC that large cardinals don't exist.

Some Common Large Cardinals

Inaccessible
Mahlo
Weakly Compact
Ramsey
Measurable
Strongly Compact
Supercompact
Huge
Superhuge
Super-n-huge for every n

The Problem of Large Cardinals

The Problem:

1. Determine which large cardinals really "exist"
2. Find a way to expand the axioms of ZFC so these "legitimate" large cardinals can be derived.

To solve these problems, it is necessary to have a deep insight into the structure of V, the wholeness of mathematics.

Insight from Maharishi Vedic Science

- Expect large cardinals to emerge from transformational dynamics of wholeness
- In mathematical language,
  - wholeness = V
  - transformation = function
- Therefore, seek a function from V to V
  - that preserves the internal structure of V
  - whose characteristics are "present at every point"

First Insight: A Global Axiom of Infinity

The work of William Lawvere (1969) shows that the Axiom of Infinity is equivalent to the existence of a map j: V -> V, where

- j = G o F
- G, F are *functors*
- G and F have strong preservation properties

Observation: We have an indication of a structure-preserving transformation. We want to give full mathematical expression to this notion. Attempt to give j itself preservation properties.

Second Insight: Making j *Exact*

Trnkova-Blass (1974) showed that existence of a nontrivial *exact* j: V->V is equivalent to existence of a *measurable cardinal.*

(Exactness is a strong preservation property)

<u>Question</u> What if we require j to preserve the entire structure of V (rather than just *some* of that structure)?

A Solution to the Problem of Large Cardinals

- The **Wholeness Axiom** (WA) asserts that there is a nontrivial elementary embedding $j : V \to V$ with the property that $j \upharpoonright X$ belongs to V whenever X belongs to V.
- **Theorem.** Assume WA and let $j : V \to V$ be the WA-embedding. Then the first cardinal κ moved by j (so that $j(\kappa) \neq \kappa$) is super-n-huge for every n (and more).
- The cardinal κ is obtained as a collapse of V to a point, to the first hint of transformation, analogous to the collapse of A to K.
- This is a solution to the Problem of Large Cardinals

Progress in Mathematical Analysis of the Infinite

Main Point

A solution to the Problem of Large Cardinals requires a deeper insight into the structure and characteristics of the mathematical universe V. Maharishi Vedic Science makes such an insight available in its description of the self-interacting dynamics of wholeness. Translating key points of this description into a mathematical axiom – the Wholeness Axiom – results in a solution to the Problem of Large Cardinals. These key points are:

- Creation emerges in the self-interacting dynamics of wholeness
- In the move of wholeness within itself, wholeness remains wholeness, unchanged by its own transformations
- The self-interacting dynamics of wholeness, though unmanifest, are present at every point in creation.
- In the move of wholeness, unbounded infinite silence, represented by A, collapses to its point value, K, which is a point of infinite dynamism, from which the creation sequentially unfolds

Q & A:
The Universe V As Wholeness - 1

- *all possibilities* All models of every mathematical theory are located in *V*. All sets needed for the development of any mathematical theory are located in *V*.

- *omniscience* Every mathematical fact is true in the model *V*. Thus, if one could view mathematics from the vantage point of *V*, the wholeness underlying mathematics, every mathematical truth could be known.

- *freedom* The power set axiom freely generates the set of all subsets of a given set. Since no restriction is placed on the sets generated in this way, the continuum may consistently be taken to have arbitrarily large cardinality.

- *unmanifest* *V* is too large to be an individual set; although all properties of sets can be rigorously determined and demonstrated using the axioms of set theory, nothing can be directly proven about *V*.

Q & A: The Universe V As Wholeness - 2

- *simplicity* A single elegant recursive rule is at the basis of the sequential and simultaneous unfoldment of all stages of the universe.

- *omnipotence* Any mathematical truth that has ever been demonstrated can be seen as a derivation from the axioms of set theory using rules of logic, and all of these can be found in coded form within the structure of the universe itself.

- *total potential of natural law* The laws governing a mathematical theory are expressed by axioms. The content of every axiom of set theory is fully realized in the universe of sets.

- *discriminating* The sets which emerge in the cumulative construction of *V* do not lead to any known paradox.

Q & A: The Universe V As Wholeness - 3

- *infinite silence* At limit stages of the construction of the universe, no new sets are added; this silent phase of the construction creates smoothness and uniformity in the universe.

- *infinite dynamism* In the construction of *V*, each new stage produced by the power set operator is larger than the previous stage; in particular, the power set operator produces an endless sequence of ever larger infinities.

- *pure knowledge* The information content in ZFC is the basis for essentially all known mathematical theorems.

- *infinite organizing power* The organizing power of a mathematical theory is expressed by its models. The models of set theory are infinite, complete, and all-inclusive.

Q & A:
The Universe V As Wholeness - 4

- *perfect orderliness* All theorems of set theory, and hence of all of mathematics, can in principle be generated automatically by a computer once sufficiently many axioms have been input.

- *self-sufficiency* All the information needed to construct the stages of the universe is coded in the first few stages of the universe; the universe can therefore reproduce itself.

- *purifying* The recursive construction of *V* systematically prevents the entry of paradoxical sets.

- *infinite creativity* All the creativity of the brightest mathematicians of recorded history can be coded up as formal theorems derivable from the simple axioms of set theory.

Q & A:
The Universe V As Wholeness - 5

- *integrating* All mathematical theories, with their own special mathematical languages, find a common basis in set theory; the interrelationships between theories are thereby more easily identified.

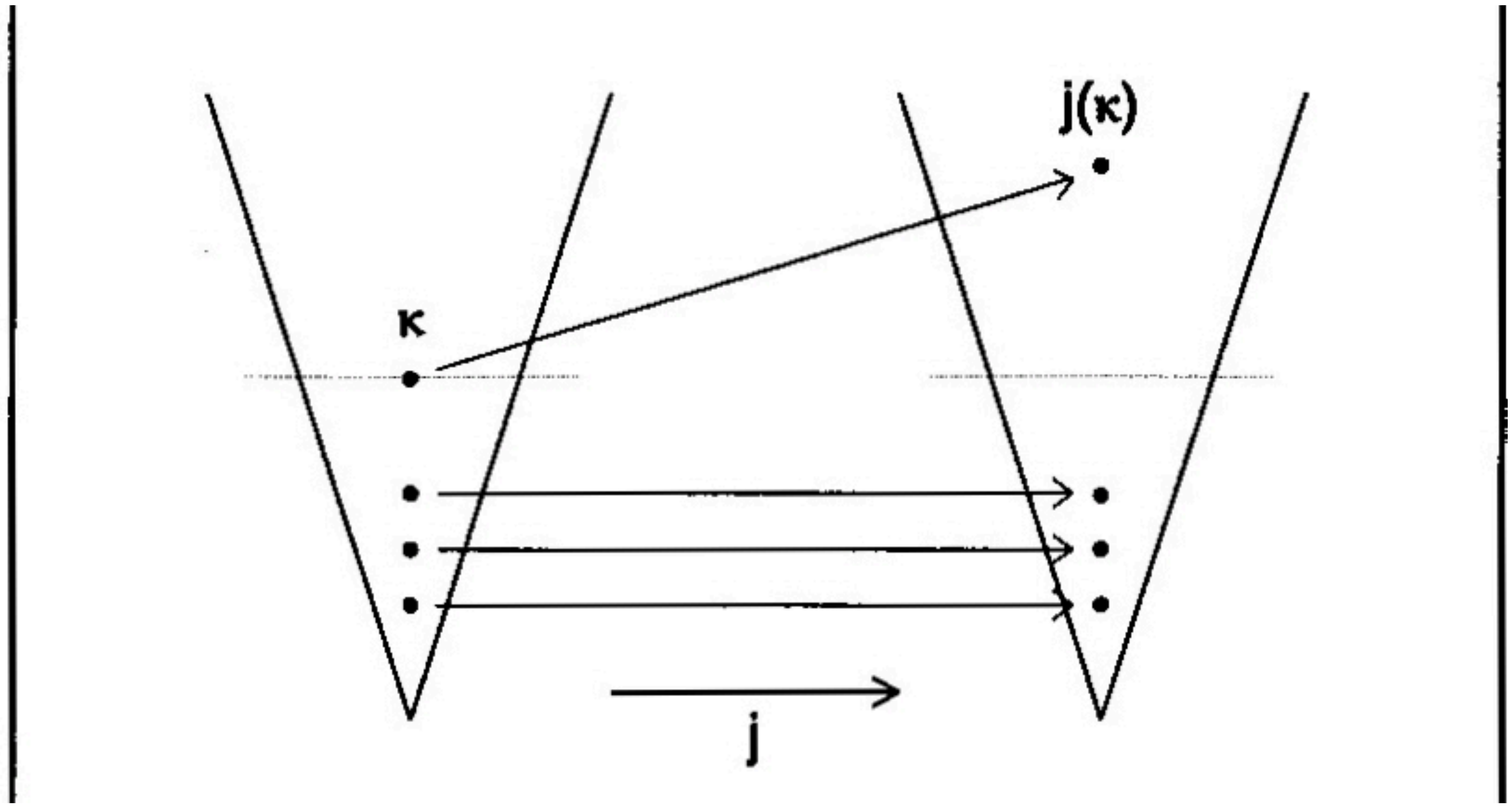
- *harmonizing* Superficial differences in style between different theories are stripped away when the formal content of these theories is expressed in the language of set theory.

- *perfect balance* Despite the differences in style and content between different theories and their models, all such models naturally emerge in the uniform and simply defined unfoldment of the stages of the universe.

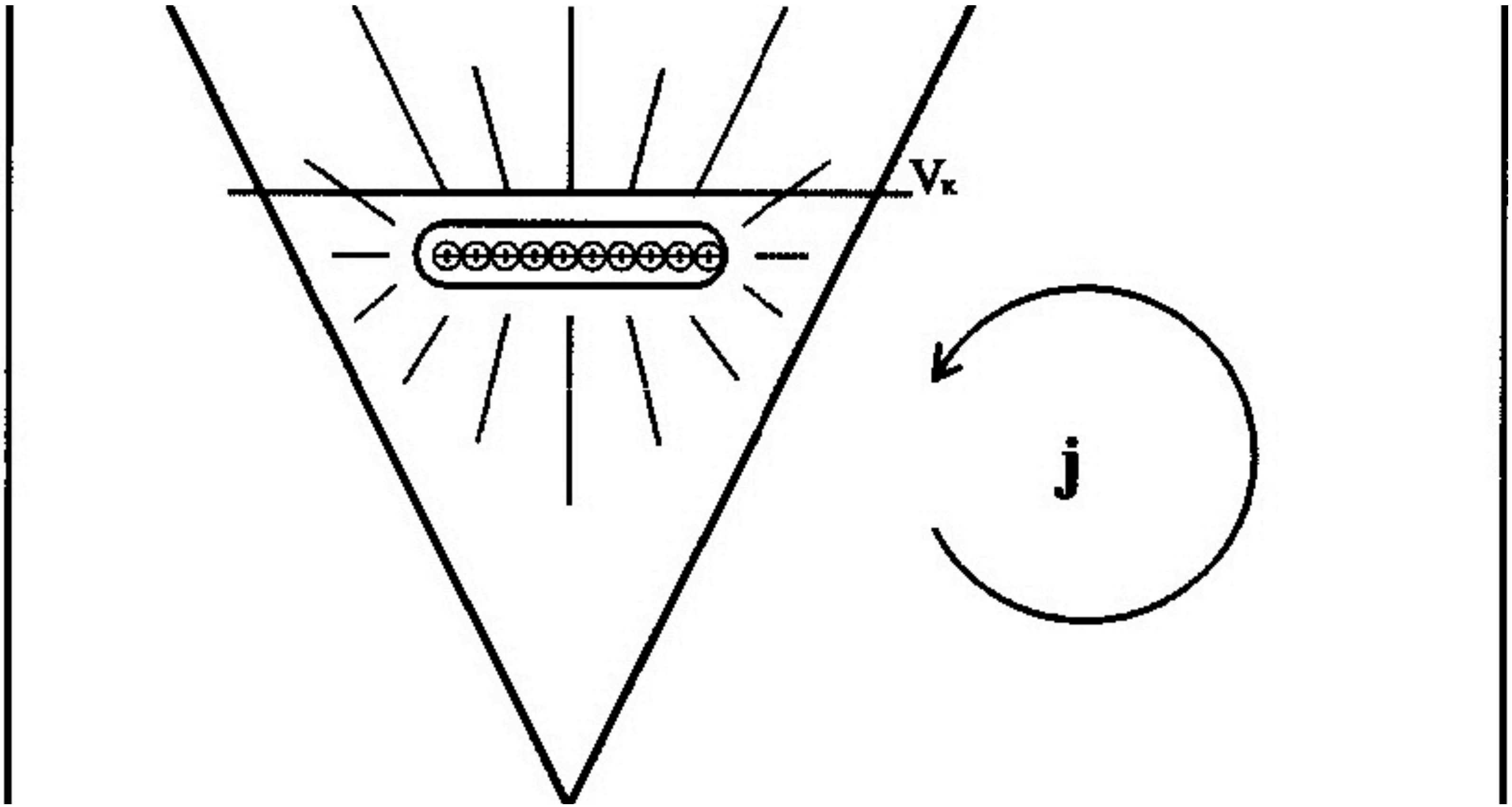
- *unboundedness* The sequence of stages of the universe *V* unfold without bound; the resulting universe *V* is so vast that it cannot be considered a set.

- *omnipresence* All mathematical structures can be located inside *V*.

Q&A: j: V -> V and Collapse of Unboundedness to a Point



Q&A:
From κ to Laver Sequence to All Sets - 1



Q&A: From κ to Laver Sequence to All Sets - 2

- **Step 1**: In the dynamics of j, κ is located
- **Step 2**: Inherent in j are derived embeddings which collectively create a κ-sequence – a *Laver sequence S*– which lives within V_κ.
- **Step 3:** From S all sets in the universe can be located. For each x there is a derived embedding i so that

  $$x = (i(S))\kappa$$

- (Mathematics) j -> κ -> Laver sequence -> all sets
  (MVS) Move within A -> K -> Veda -> the manifest universe

Q&A: Awakening From
Ignorance to TC to Unity to Brahman

- Step 1: (19^{th} century mathematics) "Nothing infinite exists"
- Step 2: A taste of the infinite (**N** exists)
- Step 3: The infinite has different sizes and expands within itself – it has a *nature*
- Step 4: "Large cardinals are an illusion"
- Step 5 : Some large cardinals seem to be real, legitimate
- Step 6: (WA) Large cardinals arise from wholeness interacting with itself (Unity)
- Step 7: (WA) *Almost all cardinals are large cardinals* (Brahman)

Q&A:
8-Fold Collapse of Infinity to a Point - 1

8-fold Prakriti	**Rishi:** A universe of mathematics	**Devata**: embeddings of the universe	**Chhandas**: point value of an embedding
prithivi (earth)	V is a model of ZFC and for almost all $\lambda < \kappa$, V_λ is a model of ZFC	There is a $\lambda < \kappa$ such that the inclusion map $\lambda \to \kappa$ is an elementary embedding	**κ** is inaccessible
jala (water)	$V \neq L$	There is an elementary embedding $j: L \to L$	0# exists
agni (fire)	$V \neq K$	There is an elementary embedding $j: V \to M$ Also, there is an exact functor $j: V \to V$	There is a measurable cardinal
vayu (air)	$V \neq L[A]$ for any A / existence of Laver sequences	For each λ, there is an inner model M and an elementary embedding $j: V \to M$ with V_λ subset of M.	There is a strong cardinal

Q&A:
8-Fold Collapse of Infinity to a Point - 2

akasha (space)	$NS_{\omega 1}$ is ω_2-saturated	For all subsets A of V_δ, there are arbitrarily large $\kappa < \delta$ such that for all $\lambda < \delta$, there is an elementary embedding j: V-> M with crit point κ, $j(\kappa) > \lambda$, V_λ subset of M, and so that A, j(A) are equal up to V_λ	There is a Woodin cardinal δ
manas (mind)	AD holds in L(R)	There are cardinals $\delta_0 < \delta_1 < \delta_2 < \ldots$ with corresponding embeddings j_0 j_1 j_2 making each δ_i Woodin and a measurable cardinal above them all	There are infinitely many Woodin cardinals with a measurable above
buddhi (intellect)	Almost all cardinals are large cardinals	For each λ, there is an elementary embedding $j: V_{\kappa+\lambda} \to V_\xi$ with critical point κ	There is an extendible cardinal
ahamkar (ego)	There are arbitrarily close approximations to an ω-huge cardinal	For each n, there is a $j_n: V \to M_n$ such that M_n is closed under n sequences.	For each n there is an n-huge cardinal.